STUDY OF HALF WAVE RESONANCES FOR THE SI ENGINEER

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Contents

Motivation

When hearing or seeing something we understand, we feel good; if not, we feel uneasy. That is exactly what happens when we look at a well or badly behaved S-parameter.

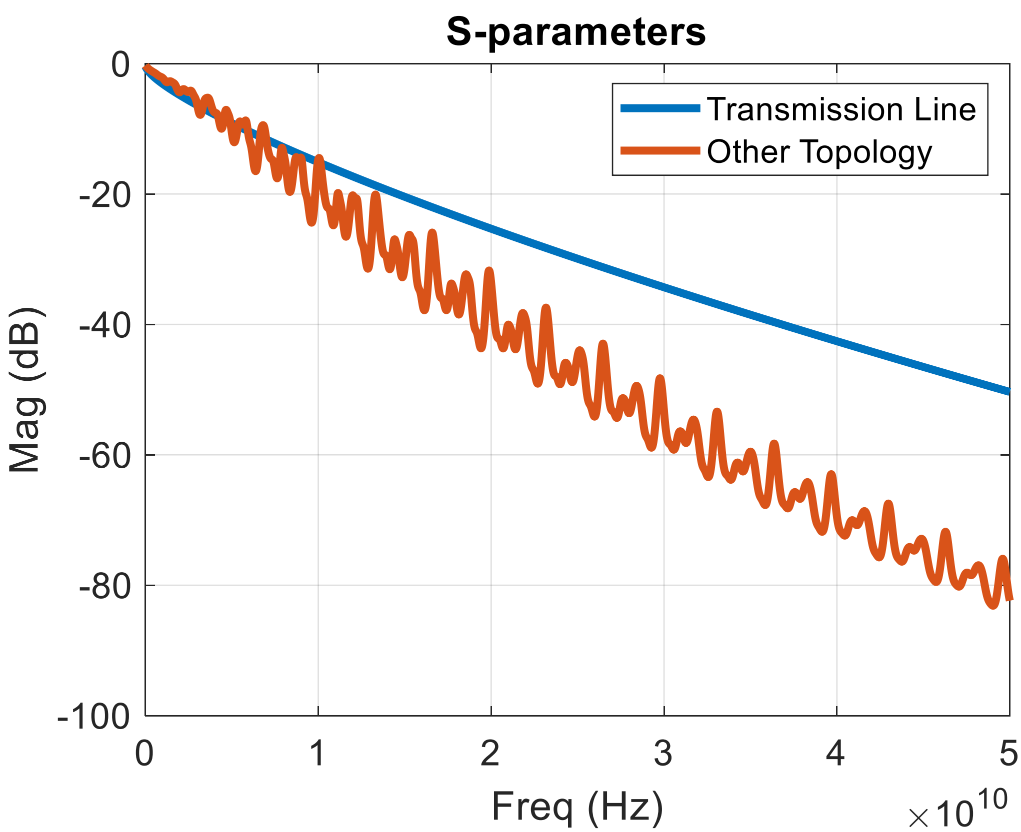


Figure 1:Transmission Line vs. Complex Topology S-parameter

The smooth line in Figure 1 makes us feel good. We have a sense of knowledge of how it would behave: for example, the longer it is, the more losses it will have, and we strongly suspect the smoothness of the curve is an indication that the line is well matched, etc.

On the other hand, the jagged line in Figure 1 with all those wiggles is not readily understandable. When looking at the curve carefully, we see two distinct, but defined periodicities that we do not yet quite understand. These types of curves give us headaches.

That ugly curve and all the resonances or periodicities seen in it are mostly due to half-wave resonances. Half-wave resonances are one of the most common and pernicious resonances encountered in topologies, but many times they go unnoticed or are ignored.

I believe it is important for the signal integrity engineer to be able to fully understand, identify, and address these resonances in different topologies.

Let me tell you how this article is organized:

I will start with a simple definition and the topological elements required for a resonance to develop, followed with frequency domain analysis. Unfortunately, as much as I would like to avoid this, I will have to throw some theory with math in here, but I will try my best to make it clear and simple. In my world view, I feel the best way to understand a particular concept is when you see it right in front of you in a simple formula, and in the case of half-wave resonances, this happens several times in the frequency-domain analysis.

In the frequency domain, I will go through a few cases to illustrate different theoretical points, some that you might normally find in books. I will relate the formulation of the analysis to what you should or should not expect to see in S-parameters.

After the frequency domain, I will transition to the time domain, and do a similar analysis. Since this section comes after all the frequency domain analysis formulation, it will be a little lighter on math, but it will rather be more visual, and I hope very easily understandable. At the end of this section, I will relate the frequency and time domain, and summarize simple relationships that can aid to approximately identify the values of these resonances in real topologies.

Then, I'll show some subtleties of half-wave resonance that will depend on the type of discontinuities at the end and show how reactive termination will act differently than resistive termination.

Finally, we'll bring all the knowledge acquired though the text, and we'll apply it to some real topologies. I will provide two examples with highly practical application relevance for the SI engineer.

Frequency domain analysis

Let us start with a definition of resonance and the elements required for it to develop.

A resonance **is the interaction between at least two discontinuities of any kind separated by an electrical length.** Let us examine this definition starting from the end:

- **Electrical Length:** For something to resonate, separation is needed. The distance between two discontinuities in the electrical world is relative. Let's consider a one-inch transmission line on a standard PCB with a nominal propagation delay of 160 ps/in. and let's ask if this is enough separation. As many things in SI, the answer is**, it depends**. For example, at 100 GHz, with a wavelength of λ = 62.5 mils, a one-inch transmission line is a big separation; we will have 16 full cycles traveling on the transmission line at any point of time. On the other hand, if we consider a frequency of 100 KHz, with a wavelength λ =62500 inches, a one-inch transmission line is just a small fraction of the wavelength, and the circuit (separation between discontinuities) could be considered lumped. In other words, there is no need to include the transmission line for calculations. The term electrical length captures this distinction.
- **Discontinuities:** After having an electrical separation, waveforms need to bounce back and forth, hence discontinuities are required at the end points for this to happen. With perfect matched loads, no reflections will happen and there will not be a resonance. The other important word in the definition is **"two."** In order to have a resonance there needs to be at least two of these discontinuities. One discontinuity will be sufficient to generate a non-ideal transmission, but not sufficient to generate a resonance with its associated dip in the insertion loss profile.
- **Interaction:** The reason a resonance develops is due to the interaction between these discontinuities. In other words, the reflection at one end should be seen at the other end after a finite amount of time. This is key for the resonance to develop. As will be seen later, different type of discontinuities at the ends of the transmission line will generate different type of resonances and standing wave patterns. In practical terms, this means the attenuation along the interconnect should not be very big, otherwise the wave will not bounce back and forth.

I will start with a very quick review of phasors which is simply a way to represent vectors that makes the whole transmission line formulation much more elegant.

Phasors

Oscillations are all around us: your heartbeat, the rotation of the earth around the sun, the pendulum of a clock in time and space. The more you look, the more you realize the vast number of examples where oscillatory behavior is present as an inherent part of the world. With that in mind it would be good to develop a method to represent oscillatory behavior in its most fundamental form with sinusoidal functions and represent it elegantly. Phasors come to the rescue.

A phasor is a complex number representing a sinusoidal function with angular frequency ω, amplitude A and initial phase ϕ.

A simple sinusoidal source, either current or voltage can be written as follows V = A · sin (2πf + ϕ), where A is the signal amplitude, f is the frequency in Hz [cycles/sec] and ϕ is the initial angle or phase.

If we imagine a vector rotating around its origin, every time the vector completes a revolution, it has traveled an angle of 2π or 360^{o}. In a linear scale, we represent frequency in units of Hz= cycles/sec, so one cycle = 2π. When we represent this as a vector rotating around its axis, in angular form, this is equivalent to ω = 2πf [rad/sec], meaning when f = 1 cycle => ω = 2π, one revolution.

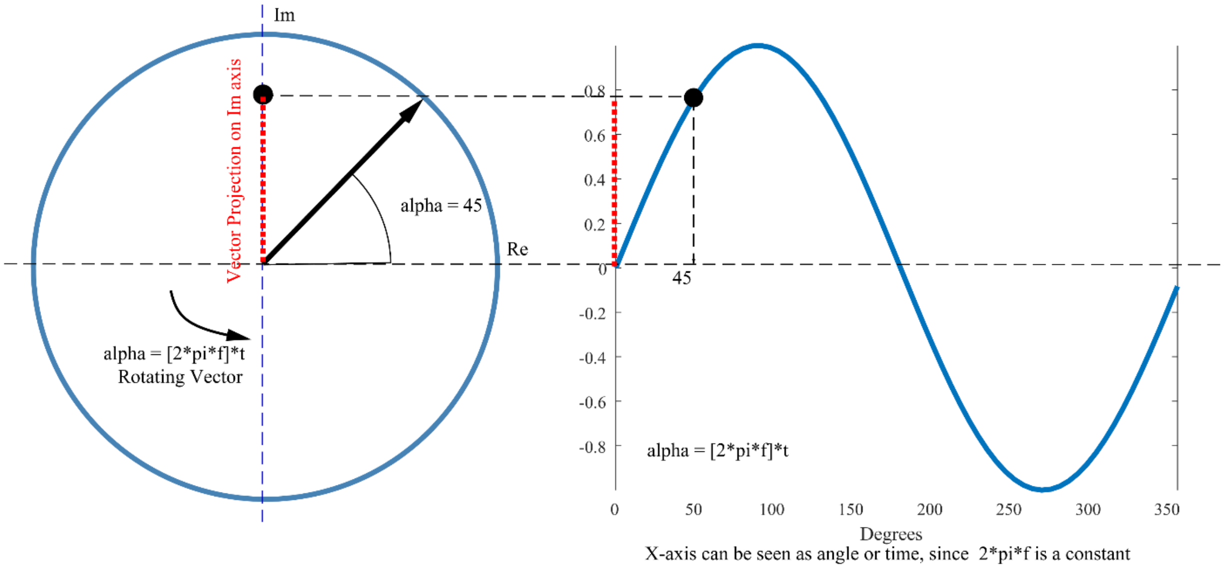


Figure 2: Phasor Representation

Imagine a vector rotating at a constant speed 2πf, if you project that rotation over the "x" or "y" axis, you will get either cos (2πf · t) or sin (2πf · t) function, respectively. Let us now imagine we have a voltage source that has an initial phase φ(z), and a particular frequency f. The fact φ(z) is dependent of position z, is irrelevant, for now imagine this is a magic voltage source, that if you move it to a different position, its initial phase changes.

$$V\ (t,\varphi) = A\ \cdot \cos\big(2\pi f\ \cdot\ t\ +\ \varphi(z)\big) \qquad 1$$

And remembering Euler's identity:

$$e^{ix} = \cos(x) + i \cdot sin(x) \qquad 2$$

So, we can say:

$$V\big(t,\varphi(z)\big) = \mathbb{R}_e\big(A \cdot e^{i(2\pi f t+\varphi(z))}\big) \qquad 3$$

$$\mathrm{V}\big(t,\varphi(z)\big) = \mathbb{R}_e\big(A \cdot e^{i(2\pi f t)} \cdot e^{i(\varphi(z))}\big) = \mathbb{R}_e\big(\widetilde{A}\ \cdot e^{i(\varphi(z))}\big) \qquad 4$$

Where the phasor $\widetilde{A}$ defined with a tilde on top is:

$$\widetilde{A} = \mathbb{R}_e\left(A \cdot e^{i(2\pi f t)}\right) \quad 5$$

Finally, the voltage source shown on 1 can be more succinctly represented as:

$$V\big(\varphi(z)\big) = \widetilde{A} \cdot e^{i\varphi(z)} \quad 6$$

Yeah, I know, the $\mathbb{R}_e$ has been omitted and I do not like it either, but that is common in all the literature, meaning a priori it is known we will be taking only the real part, so we do not put it in to begin with. In any event, we see how nicely and succinctly we have written the equation for the source and, in addition we have hidden the voltage source time dependency inside the phasor $\widetilde{A}$.

Please also note that equally as easy, we could have defined the phasor, with dependency on $\varphi(z)$ such the source was only dependent on time, that is an equally valid phasor representation.

Brief Review of the Wave Equation Solution

I will spare you the derivation of Maxwell equations to get to the Telegrapher's equations, since that can be found pretty much in every EM book. To highlight some useful points, I will start from the solutions of the Telegrapher's equation. For this study, please imagine a wave on a lossless transmission line propagating in the z direction with characteristic impedance Zc

$$v(z,t) = A \cdot \cos(wt - \beta z) + B \cdot \cos(wt + \beta z) = \mathbb{R}_e\left(A \cdot e^{-i(\beta z - \omega t)} + B \cdot e^{i(\beta z + \omega t)}\right)$$

$$i(z,t) = \frac{A}{Z_c} \cdot \cos(wt - \beta z) - \frac{B}{Z_c} \cdot \cos(wt + \beta z) = \mathbb{R}_e\left(\frac{A}{Z_c} \cdot e^{-i(\beta z - \omega t)} - \frac{B}{Z_c} \cdot e^{i(\beta z + \omega t)}\right)$$

$$\beta = \omega \cdot \sqrt{L \cdot C} \leftarrow \text{Lossless propagation constant}$$

$$\lambda = \frac{2\pi}{\beta} \leftarrow \text{Wavelength}$$

$$A, B \leftarrow \text{In the simplest (real) case, Voltage/Current magnitudes}$$

$$A \cdot cos\left(\omega t - \beta z\right), \frac{A}{Z_C} \cdot cos\left(\omega t - \beta z\right) \leftarrow \text{Forward propagating, voltage and current waves}$$

$$B \cdot cos(\omega t + \beta z), \frac{B}{Z_C} \cdot cos(\omega t + \beta z) \leftarrow \text{Backward propagating voltage and current waves}$$

$$A \cdot cos\left(\omega t_1 - \beta z_1\right) = A \cdot cos\left(\omega t_2 - \beta z_2\right) \leftarrow \text{Same magnitude as the waveform travels}$$

$$(\omega t_1 - \beta z_1) = (\omega t_2 - \beta z_2)$$

$$\frac{\omega}{\beta} = \frac{z_2 - z_1}{t_2 - t_1}$$

$$\frac{\omega}{\beta} > 0, t_2 > t_1 \implies z_2 > z_1 \text{meaning the waveform is propagating from left to right}$$

$$B \cdot cos\left(\omega t_1 + \beta z_1\right) = B \cdot cos\left(\omega t_2 + \beta z_2\right) \leftarrow \text{Same magnitude as the waveform travels}$$

$$(\omega t_1 + \beta z_1) = (\omega t_2 + \beta z_2)$$

$$\frac{-\omega}{\beta} = \frac{z_2 - z_1}{t_2 - t_1}$$

$$\frac{\omega}{\beta} > 0, t_2 > t_1 \implies z_2 < z_1 \text{meaning the waveform is propagating from right to left}$$

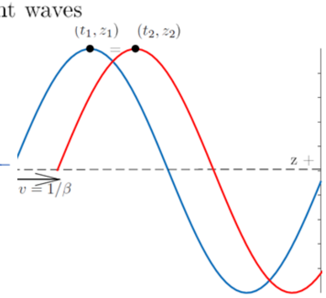


Figure 3: Wave Solution Equations

Note the solution contains two waves, one propagating in the forward and one propagating in the backward direction. Following the above figure, the propagation direction can be easily understood.

Another important parameter that will prove useful in our calculations is the input impedance Z_{in}. This is the impedance seen at the input of a transmission line including reflections. If we just divide the voltage/current wave solution above and perform some math magic, we will get

$$Zin(z) = Zc \cdot \frac{Z_L + i \cdot Z_C \cdot \tan(\beta z)}{Z_C + i \cdot Z_L \cdot \tan(\beta z)} = Zc \cdot \frac{e^{i\beta z} + \Gamma_L \cdot e^{-i\beta z}}{e^{i\beta z} - \Gamma_L \cdot e^{-i\beta z}} \quad 7$$

Γ_L is the reflection coefficient at the load. Note the conceptual difference between Z_c *and* Z_{in}.The characteristic impedance, Z_c is the ratio of voltage and currents of only **one** wave propagating through the transmission line at every point. On the other hand, Z_{in} is the ratio of voltage and current of **all** the forward and backward superimposed waves at a **single** location, in this case the input.

We can represent the wave equation solution of Figure 3 in phasor form as follows:

By defining the phasors by a $\sim$:

$$\tilde{V}_a = A \cdot e^{i\omega t} \text{ and } \tilde{V}_b = B \cdot e^{i\omega t} \quad 8$$

Furthermore, we can say:

$$\tilde{V}^+ = \tilde{V}_a \cdot e^{-i\beta z} \text{ and } \tilde{V}^- = \tilde{V}_b \cdot e^{i\beta z} \quad 9$$

I can then simply represent the wave solution as follows:

$$V(z) = \tilde{V}_a \cdot e^{-i\beta z} + \tilde{V}_b \cdot e^{i\beta z} = \tilde{V}^+ + \tilde{V}^- \quad 10$$

This formula nicely allows us to define the reflection coefficient Γ as:

$$\Gamma = \frac{\tilde{V}^-}{\tilde{V}^+} = \frac{\tilde{V}_b \cdot e^{i\beta z}}{\tilde{V}_a \cdot e^{-i\beta z}} = \frac{B}{A} \cdot e^{i2\beta z} \xRightarrow[z=0=load\ location]{} \Gamma_L = \frac{B}{A} \quad 11$$

From 10 and 11 we can say:

$$V(z) = \tilde{V}^+ \cdot (1 + \Gamma_L) = \tilde{V}_a \cdot (e^{-i\beta z} + \Gamma_L \cdot e^{i\beta z}) \quad 12$$

Equation 11 is great because it shows the reflection coefficient definition clearly, when considering the reference plane (i.e., z=0) at the load location (end of the line)

Let us pause a bit here, and take note of a few subtleties:

1. The $\mathbb{R}e$ is not shown as mentioned previously, but remember, it is there (omitted for simplicity)

2. The fact that in the TEM mode the electric and magnetic field are perpendicular to the propagation direction, makes the propagation and the frequency(time) dependency of the fields independent.

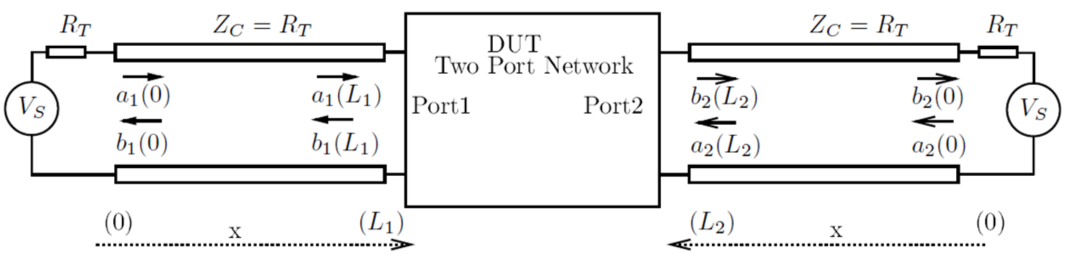
So, where are we?

We have shown the solution to the Telegrapher's equation can have two traveling waves, one propagating in the positive direction, from source to load represented by $\tilde{V}_a \cdot e^{-i\beta z}$, and one traveling in the backward direction from load to source represented by $\tilde{V}_b \cdot e^{i\beta z}$. The magnitude of the backward propagating wave will depend on reflections that can be calculated from our old friend the reflection coefficient at the load Γ_L. We've also learned that each propagating wave will have a time dependency $\tilde{V}_a = A \cdot e^{i\omega t}$ and $\tilde{V}_b = B \cdot e^{i\omega t}$ due to the excitation that is independent of the perpendicular propagation of the wave.

With that in mind after a short review of S-parameter definition we can start analyzing the resonance case and link it to what we can expect in the S-parameter frequency domain.

S-Parameter Definition Review

In order to meet the conditions in Figure 4, there should not be a reflection at the $x = 0$, meaning the transmission line must be perfectly matched to the source and/or load $a|b_{1|2}\,(0)$. Since $a(x)$, $b(x)$ are dependent directly on the traveling voltage and current waveforms and because we know at the measurement point there will not be any reflections, S-parameters can be directly calculated by a ratio of voltages with respect to an ideal transfer.



$$S_{11} = \frac{b_1(L_1)}{a_1(L_1)}\Big|_{a_2(L_2)=0} \qquad a_1(x) = \frac{\tilde{V_1}^{+}}{\sqrt{Z_C}}, b_1(x) = \frac{\tilde{V_1}^{-}}{\sqrt{Z_C}}$$

$$S_{21} = \frac{b_2(L_2)}{a_1(L_1)}\Big|_{a_2(L_2)=0}$$

$$S_{12} = \frac{b_1(L_1)}{a_2(L_2)}\Big|_{a_1(L_1)=0} \qquad a_2(x) = \frac{\tilde{V_2}^{+}}{\sqrt{Z_C}}, b_2(x) = \frac{\tilde{V_2}^{-}}{\sqrt{Z_C}}$$

$$S_{22} = \frac{b_2(L_2)}{a_2(L_2)}\Big|_{a_1(L_1)=0}$$

Figure 4: S-Parameter Definition Diagram

The normalized waves, $a(x), b(x)$, have been defined as functions of phasors previously introduced, so we are in luck, and we can start complicating topologies to get resonances and relate the voltages to the

S-parameters. Before we get into cases involving transmission lines, let's calculate the S-parameter of a shunt 50 Ohms resistor.

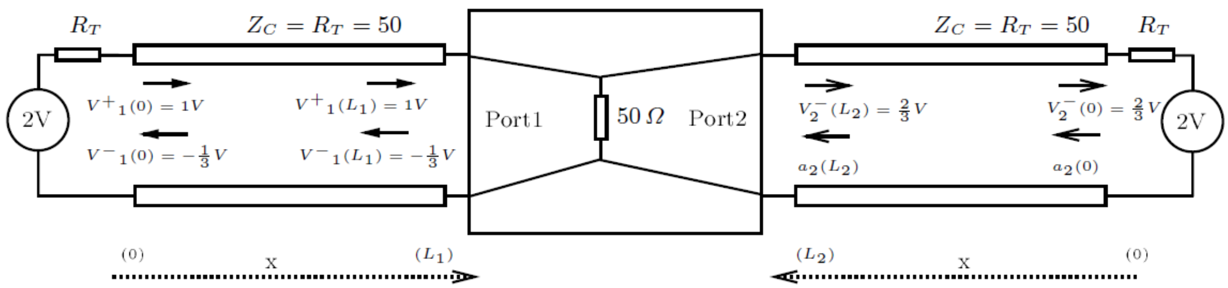


Figure 5: S-parameter 50 Ohms Shunt Diagram

Let us drive from port 1 first and find $S_{11}(dB), S_{21}(dB)$

$$S_{11} = \frac{b_1(L_1)}{a_1(L_1)} = \frac{\tilde{V}_1^-(L_1)}{\tilde{V}_1^+(L_1)} = -\frac{1}{3} \underset{dB}{\Rightarrow} S_{11}(dB) = 20 \cdot \log_{10}\left|-\frac{1}{3}\right| = -9.54dB \quad 13$$

$$S_{21} = \frac{b_2(L_2)}{a_1(L_1)} = \frac{\tilde{V}_2^-(L_2)}{\tilde{V}_1^+(L_1)} = \frac{2}{3} \underset{dB}{\Rightarrow} S_{21}(dB) = 20 \cdot \log_{10}\left|\frac{2}{3}\right| = -3.52dB \quad 14$$

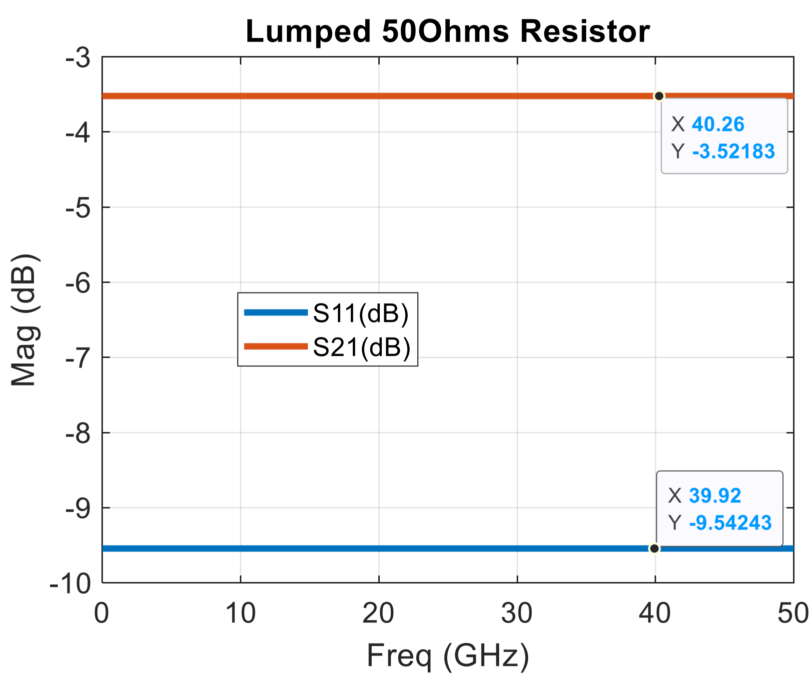


Figure 6: S-parameter 50 Ohms Shunt Results

Matched Case ($Z_S = Z_C = Z_L$)

This is the simplest but most boring case since there will be only one propagating wave traveling on the transmission line. Following previous definitions:

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = 0, Z_{in} = Z_C \qquad 15$$

$$\tilde{V}_a \cdot (e^{-i\beta z} + \Gamma_L \cdot e^{i\beta z}) = \tilde{V}_a \cdot e^{-i\beta z} \qquad 16$$

$$v(z,t) = V \cdot \cos(\omega t - \beta z) \qquad 17$$

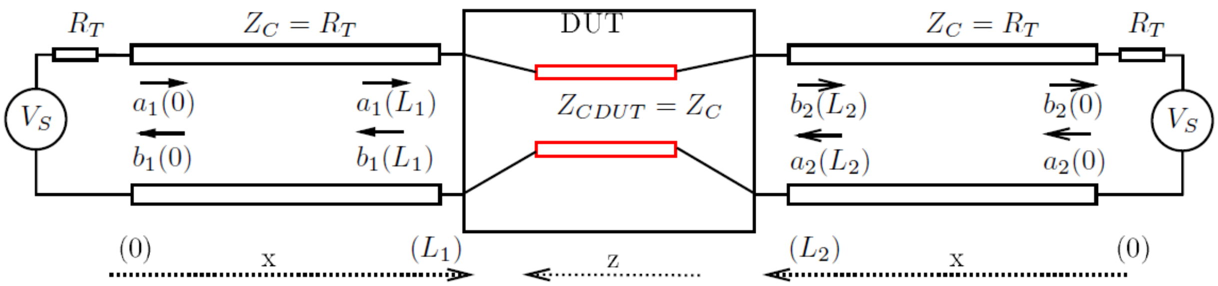


Figure 7: Matched Case S-parameter Diagram

As we can see in 15 the reflection coefficient at the load is zero, meaning the input impedance in the line is identical to Z_C , this in turns means that there are no reflections and hence we are left with only one propagating wave from source to load. In term of S-parameters, just replacing our DUT by a perfectly matched transmission line as shown in Figure 7, yield:

$$S_{11} = \frac{b_1(L_1)}{a_1(L_1)} = \frac{\tilde{V}_1^-(L_1)}{\tilde{V}_1^+(L_1)} = 0 \underset{dB}{\Rightarrow} S_{11}(dB) = 20 \cdot \log_{10}|0| = -\infty\ dB \qquad 18$$

$$S_{21} = \frac{b_2(L_2)}{a_1(L_1)} = \frac{\tilde{V}_2^-(L_2)}{\tilde{V}_1^+(L_1)} = 1 \underset{dB}{\Rightarrow} S_{21}(dB) = 20 \cdot \log_{10}|1| = 0\ dB \qquad 19$$

Note there will be a phase difference between $\tilde{V}_2^-(L_2)$ *and* $\tilde{V}_1^+(L_1)$ due to the transmission line delay, not analyzed here.

Also please note that since now the DUT has a delay, I have included a z axis that represents a distance within the DUT. I have chosen z just to differentiate from x that has already been used on the external transmission lines for the S-parameters.

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Let us spice up the analysis a bit and consider a DUT with a single, but strong reflection at the load side and see what happens.

Shorted Case ($Z_S = Z_C,\ Z_L = 0$)

If we grab the transmission line on the previous case, but we short it at the end, things start to get a little more interesting, since now, we are forcing a reflected wave to travel back. In this case, we are forcing a full reflection since the transmission line is shorted, meaning everything will be reflected and nothing will be transmitted.

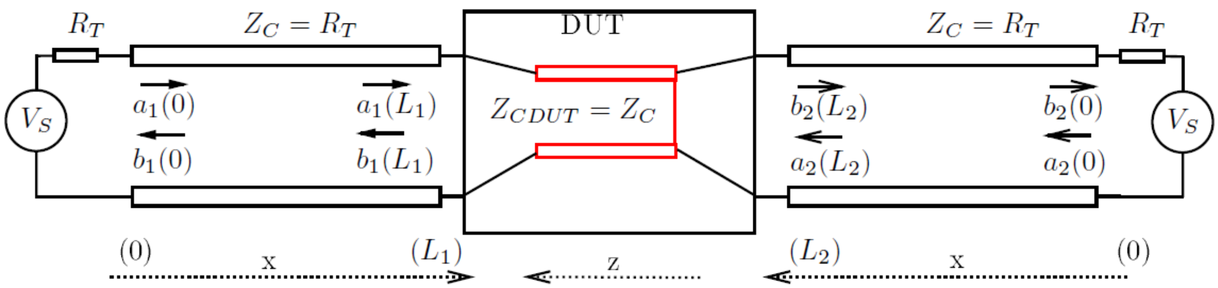

Figure 8: S-parameters, Shorted Transmission Line Diagram

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = -1, \qquad Z_{in} = Z_C \cdot \frac{e^{i\beta z} - e^{-i\beta z}}{e^{i\beta z} + e^{-i\beta z}} \qquad 20$$

$$V(z) = \tilde{V}_a \cdot (e^{-i\beta z} + \Gamma_L \cdot e^{i\beta z}) = \tilde{V}_a \cdot (e^{-i\beta z} - e^{i\beta z}) = \frac{-2i \cdot \tilde{V}_a \cdot (e^{i\beta z} - e^{-i\beta z})}{2i} \qquad 21$$

$$v(z,t) = -2i \cdot \tilde{V}_a \cdot \sin(\beta z) = e^{-i\frac{\pi}{2}} \cdot 2A \cdot e^{i\omega t} \cdot \sin(\beta z) \qquad 22$$

$$v(z,t) = \mathbb{R}_e \left(2A \cdot e^{i\left(\omega t - \frac{\pi}{2}\right)}. \sin(\beta z)\right) = 2 \cdot A \cdot \cos\left(\omega t - \frac{\pi}{2}\right) \cdot \mathbf{sin}\ (\boldsymbol{\beta z}) \qquad 23$$

Equation 23 shows something different: (1) the maximum voltage attained is 2 V, even though we are only sending 1 V. This highlights the fact we'll have reflected waveforms that will constructively add at different points in the transmission line and could duplicate the total voltage at some location, (2) the normal phasor time rotation $\cos(\omega t + \phi)$ seems to be modulated with a $\sin(\beta z)$ function that depends on z (distance). When z=0, right at the load $\sin(\beta z) = 0$ the voltage will be zero. This is expected, since we shorted the line at the load. The remarkable thing is if we move away from the load any multiples of 1/2 wavelength $z = n \cdot \frac{\lambda}{2}, since\ \beta = \frac{2\pi}{\lambda} \Rightarrow \sin(\beta z) = \sin(n\pi) = 0$, the voltage will also be exactly zero,

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meaning, the load voltage of zero value is repeating at periodical intervals in the transmission line as long as the transmission line is longer than $\frac{\lambda}{2}$. This clearly shows a standing wave is formed.

Let's take for example $z = n \cdot \frac{\lambda}{4}, since\ \beta = \frac{2\pi}{\lambda} \Rightarrow \sin(\beta z) = \sin\left(n\frac{\pi}{2}\right) = 1$, meaning at that spatial point the voltage will be at its maximum of 2 V. Recognize the voltage will not be 2 V at all times, recall we have a rotating phasor which means the instantaneous voltage at that location will change based on the $\cos\left(\omega t - \frac{\pi}{2}\right)$ function and only when it's 1, the instantaneous absolute voltage will be at the maximum of 2 V.

Imagine each transmission line point or position has a maximum voltage it can attain and no more. If for example we make $z = \frac{\pi}{8}$**,** this will result on a maximum voltage of approximately 0.75 V, meaning the time varying voltage at that point will never exceed that value. Therefore, the standing wave should be viewed as **the envelope** under which the time varying signal $A \cdot e^{i(\omega t - \frac{\pi}{2})}$ will be oscillating.

Now we can formally introduce a new metric called VSWR (voltage standing wave ratio), that is defined as the ratio of the maximum to minimum voltage on the transmission line, a value of 1 means no reflections and it's highly desirable on many conditions, a value bigger than 1 implies reflections and the formation of a standing wave. In our example, $VSWR = \infty$, represents full reflection. Figure 9 shows the VSWR of our example.

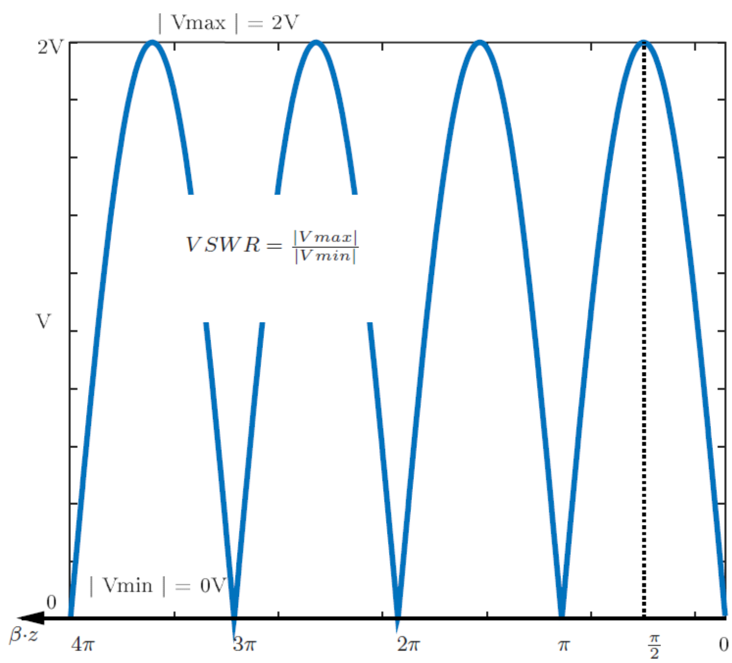

Figure 9: VSWR

Let us now calculate the S-parameters for this case.

$$S_{11} = \frac{b_1(L_1)}{a_1(L_1)} = \frac{\tilde{V}_1^-(L_1)}{\tilde{V}_1^+(L_1)} = -1 \underset{dB}{\Rightarrow} S_{11}(dB) = 20 \cdot \log_{10}|-1| = 0\ dB \quad (24)$$

$$S_{21} = \frac{b_2(L_2)}{a_1(L_1)} = \frac{\tilde{V}_2^-(L_2)}{\tilde{V}_1^+(L_1)} = 0 \underset{dB}{\Rightarrow} S_{21}(dB) = 20 \cdot \log_{10}|0| = -\infty\ dB \qquad 25$$

Even though the reflection (short) happens at port 2, the reflected wave will return unchanged in magnitude in our lossless example arriving as $\tilde{V}_1^-(L1)$ with equal magnitude and opposite phase as $\tilde{V}_1^+(L1)$ resulting on an $S_{11} = 0dB$. Equivalently for the transfer, we have a short at port 2, meaning no transfer will be possible at any frequency so $S_{21} = -\infty$

But wait, we have a full reflection in here, so where is my resonance or dips in the S-parameters?

The sometime confusing part is that since we have a standing wave, we might expect to see some sort of insertion loss dips (or resonances), but we don't. Going back to the resonance definition, we notice we would need at least two reflections or discontinuities with some separation between them for a resonance to develop. In this case we have only one discontinuity at the load since the source is perfectly matched. The single reflection is sufficient to produce a strong standing wave and VSWR, **but the presence of a standing wave does not imply a resonance**.

Resonant Case ($Z_S = Z_L = 33\Omega,\ Z_c = 50$)

Finally, we have arrived at the case with two discontinuities that should produce standing waves **and dips/peaks in the S-parameters insertion loss**.

Please take a moment to study the DUT. It's a lossless transmission line with a delay of 150ps, $Z_C = 50$ with $\frac{100}{3}\Omega$ discontinuity at each end. The termination value is the same at both ends, a condition necessary to create a half-wave resonance. The diagram has been created making the connection to the DUT with cable fixtures Z_{fix} explicit with the intention to make the circuit evident for S-parameters extraction.

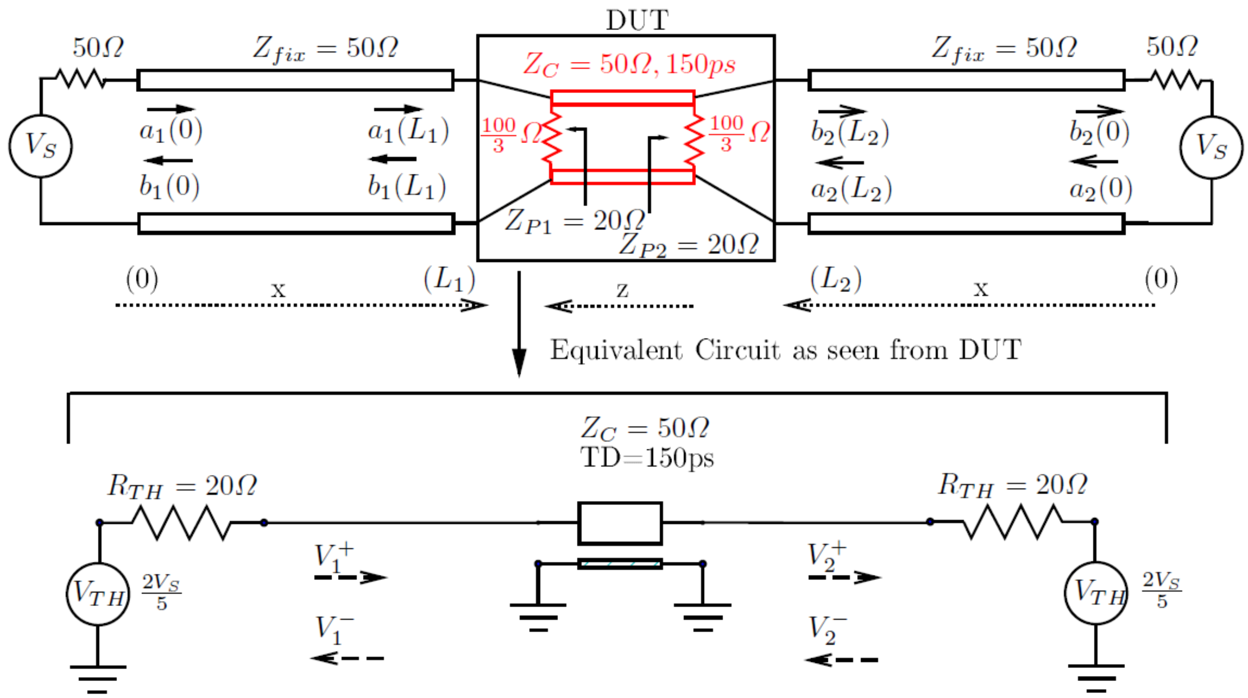

Figure 10: S-Parameters, Resonant Case Diagram

This topology meets the resonance requirements since it has two discontinuities separated by an electrical distance. It is also connected to a Vs through a cable (fixture) with a $Z_{fix} = 50\Omega$ to prepare the topology for evident S-parameter extraction.

Since this circuit is a bit more complicated, I'll simplify it by creating its Thevenin equivalent as shown in Figure 10 . The Thevenin equivalent circuit can be derived as follows:

$$V_{TH} = \frac{V_s \cdot \frac{100}{3}}{50 + \frac{100}{3}} = \frac{2}{5} Vs \quad (26)$$

$$R_{TH} = \frac{50 \cdot \frac{100}{3}}{50 + \frac{100}{3}} = 20\Omega \quad (27)$$

The easiest way to visualize this is by creating the Thevenin equivalent circuit as seen from inside the DUT toward the source or load, almost as pulling the $\frac{100}{3}\Omega$ termination resistance out of the DUT and assuming the calculation is done at DC, hence removing Z_{fix}. Please note, removing Z_{fix} in the formulation context is valid, since the only purpose of that fixture transmission line is to provide spatial separation so we can clearly see the incident and reflected waves at the DUT from the source. Since the fixture line is ideal, the only difference between the waveforms at the source/load resistor versus at the DUT input/output is simply a phase delay that can be adjusted by changing reference planes.

We will calculate the voltages and currents in the circuit at two different wavelengths $\lambda = \frac{\pi}{4}, \frac{\pi}{2}$, since with dual discontinuity we should see a resonance and expect to see different results at different wavelengths. We'll first derive the results theoretically including the S-parameters calculations, and finally we'll do a simulation to show the response over frequency.

The first step for the calculation is to determine the value of voltages and currents at port 1.

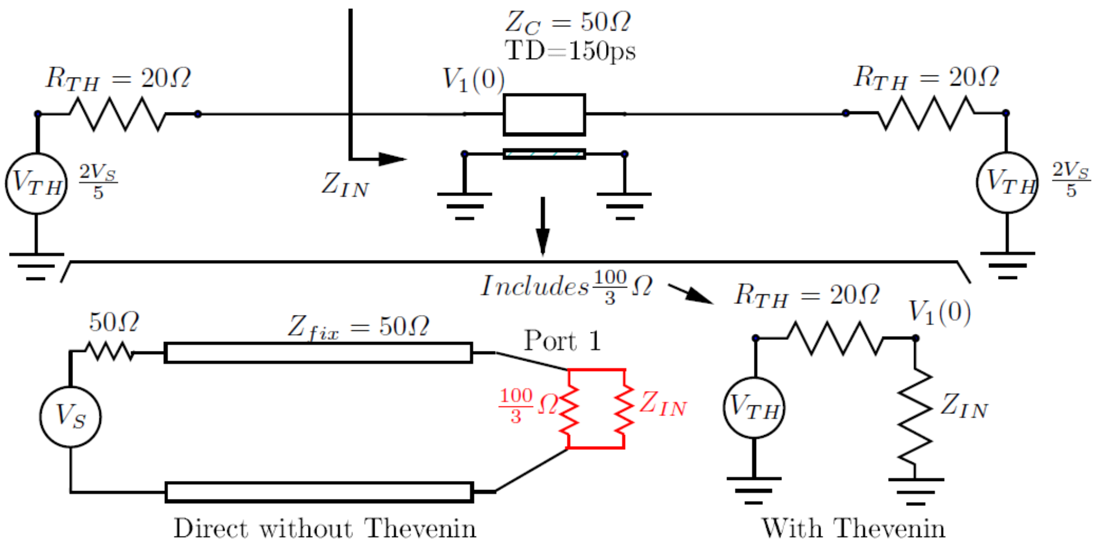

Figure 11: S-Parameters, Port 1 Voltage and Current Calculations

Recognize in Figure 11 that Z_{in} observed at the beginning of the line is not Z_c, but rather the input impedance as shown in 7. At the end of the transmission line there is a discontinuity with its corresponding reflection. The transmission line will transform the load reflection through the transmission line length in a frequency dependent way and present that reflection at the input of the DUT transmission synthesized in Z_{in}. Also please note $z = 0$ is at the output of the DUT, not at the source.

To calculate the voltage at port 1 for both wavelengths, we need Z_{in}, and thanks to the Thevenin simplification it can be calculated as follows:

$$\begin{cases} z = \frac{\lambda}{2}, Z_{in} = 50 \cdot \frac{20 + i \cdot 50 \cdot \tan(\pi)}{50 + i \cdot 20 \cdot \tan(\pi)} = 20\Omega \\ z = \frac{\lambda}{4}, Z_{in} = 50 \cdot \frac{20 + i \cdot 50 \cdot \tan\left(\frac{\pi}{2}\right)}{50 + i \cdot 20 \cdot \tan\left(\frac{\pi}{2}\right)} = 125\Omega \end{cases} \quad (28)$$

Note that $Z_{in}\left(\frac{\lambda}{2}\right) = 20\Omega$ since at half-wave resonance the DUT transmission line should reproduce the load impedance at the other end as shown in Figure 9. With these results, assuming $V_s = 2V$, we can proceed to calculate the total voltage at port 1 of the DUT.

$$\begin{cases} z=\frac{\lambda}{2}, V_1(0)=\frac{2}{5}Vs\cdot\frac{Z_{in}}{Z_{in}+R_{TH}}=\frac{2}{5}2\cdot\frac{20}{20+20}=0.4V \\ z=\frac{\lambda}{4}, V_1(0)=\frac{2}{5}Vs\cdot\frac{Z_{in}}{Z_{in}+R_{TH}}=\frac{2}{5}2\cdot\frac{125}{20+125}=0.6897V \end{cases} \quad (29)$$

The total current at the DUT transmission line input is:

$$\begin{cases} z=\frac{\lambda}{2}, I_1(0)=\frac{V_1(0)}{Z_{in}}=\frac{0.4}{20}=20mA \\ z=\frac{\lambda}{4}, I_1(0)=\frac{V_1(0)}{Z_{in}}=\frac{0.6897}{125}=5.5mA \end{cases} \quad (30)$$

So far, only the total voltages and currents have been calculated at port 1. The next step is to calculate the voltage and current waves injected into the DUT transmission line. Here is where we start to relate voltage and currents to propagating waves as expected in S-parameters.

If you refer to back to (10) you'll see that $V_{total}=\tilde{V}^{+}+\tilde{V}^{-}$ and $I_{total}=\frac{\tilde{V}^{+}-\tilde{V}^{-}}{Z_c}$, so replacing the total current and voltage we could calculate the incident and reflected waveforms at port 1 as follows:

$$\begin{cases} z=\frac{\lambda}{2}, \tilde{V}_1^{+}=\frac{V_1(0)+I_1(0)\cdot Z_c}{2}=\frac{0.4+0.02\cdot 50}{2}=0.7V \\ z=\frac{\lambda}{4}, \tilde{V}_1^{+}=\frac{V_1(0)+I_1(0)\cdot Z_c}{2}=\frac{0.6897+0.0055\cdot 50}{2}=0.4823V \end{cases} \quad (31)$$

$$\begin{cases} z=\frac{\lambda}{2}, \tilde{V}_1^{-}=\frac{V_1(0)-I_1(0)\cdot Z_c}{2}=\frac{0.4-0.02\cdot 50}{2}=-0.3V \\ z=\frac{\lambda}{4}, \tilde{V}_1^{-}=\frac{V_1(0)-I_1(0)\cdot Z_c}{2}=\frac{0.6897-0.0055\cdot 50}{2}=0.2074V \end{cases} \quad (32)$$

The propagating waves in the DUT's transmission line are shown in Figure 12

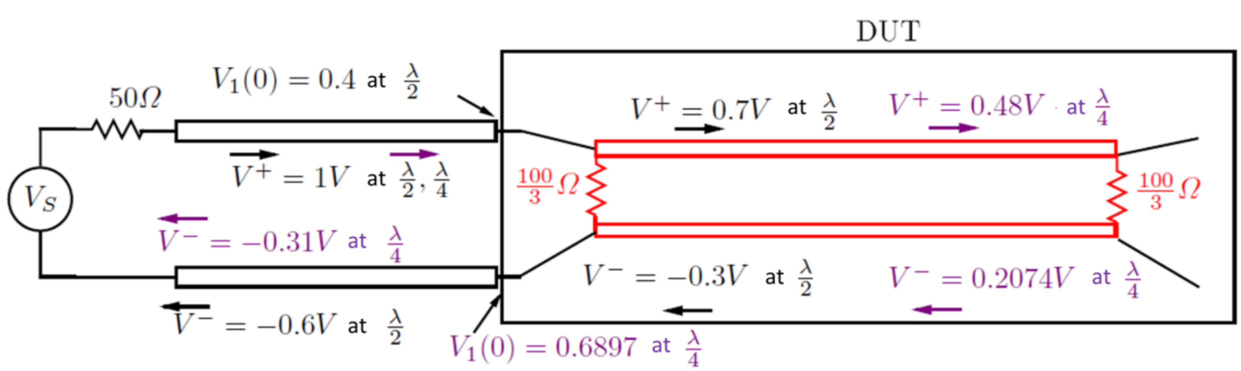

Figure 12: S-Parameters, Explicit Incident and Reflected Waves

At this point we have all the ingredients to calculate the voltage and current at port 2. We simply replace the values in the wave equation solution 10 and recalling this is a lossless system and that $\tilde{V}_1^+ = \tilde{V}_2^+ and\ \tilde{V}_2^- = \tilde{V}_1^-$ due to continuity condition we get:

$$\begin{cases} z = \frac{\lambda}{2}, V_2(L) = \tilde{V}_2^+ \cdot e^{-i\beta z} + \tilde{V}_2^- \cdot e^{i\beta z} = \tilde{V}_1^+ \cdot e^{-i\pi} + \tilde{V}_1^- \cdot e^{i\pi} = -\tilde{V}_1^+ - \tilde{V}_1^- = -0.4V \\ z = \frac{\lambda}{4}, V_2(L) = \tilde{V}_1^+ \cdot e^{-i\frac{\pi}{2}} + \tilde{V}_1^- \cdot e^{i\frac{\pi}{2}} = -j\left(-\tilde{V}_1^+ + \tilde{V}_1^-\right) = 0.2749V\angle 90° \end{cases} \quad 33$$

All the analysis above was to identify the incident and reflected waves inside the DUT. To calculate S-parameters, we need to do the same thing, but at the source and load destination, in order to denote this, all the voltages will include the subindex P.

Since we assumed a $V_S = 2V$ by following the formulas below, we can easily find the incident and reflected waves from and to the ports. Note the $\tilde{V}_{P2}^- = V_2(L)$ simply because that is the signal amplitude at the end of the DUT, that gets transmitted without reflections at port 2, which is the necessary condition for S-parameter calculation.

$$\tilde{V}_{P1}^+ = 1V, \qquad \tilde{V}_{P1}^- = \Gamma_{P1} \cdot \tilde{V}_{P1}^+, \qquad \tilde{V}_{P2}^- = V_2(L) \quad 34$$

$$Z_{P1} = \frac{\frac{100}{3} \cdot Z_{in}}{\frac{100}{3} + Z_{in}}, \Gamma_{P1} = \frac{Z_{P1} - 50\Omega}{Z_{P1} + 50\Omega}$$

$$S_{11} = \frac{b_1(0)}{a_1(0)} = \frac{\tilde{V}_{P1}^-}{\tilde{V}_{P1}^+}, S_{21} = \frac{b_2(L)}{a_1(0)} = \frac{\tilde{V}_{P2}^-}{\tilde{V}_{P1}^+}$$

Applying the formulation above and converting to dB we obtain:

$$\begin{cases} z = \frac{\lambda}{2}, \begin{cases} S_{21} = 20 \cdot \log_{10}(|0.4|) = -7.95\ dB \\ S_{11} = 20 \cdot \log_{10}(|-0.6|) = -4.43\ dB \end{cases} \\ z = \frac{\lambda}{4}, \begin{cases} S_{21} = 20 \cdot \log_{10}(|0.2749|) = -11.21\ dB \\ S_{11} = 20 \cdot \log_{10}(|-0.3103|) = -10.16\ dB \end{cases} \end{cases} \quad 35$$

I have calculated the effect of the resonance circuit at two specific lengths. Note, the DUT transmission line in question has a delay of 150ps. A frequency $f = 3.33GHz$, with a period $T = \frac{1}{3.33GHz} = 300ps$ represents two times the propagation delay. This means that the length of the line $len = \frac{\lambda}{2}$ at this frequency is half a wavelength. Equivalently it would require four of our transmission lines to fill a wavelength with a frequency of $f = 1.7GHz$ and a period of $T = 600ps$. Meaning this frequency represents the $len = \frac{\lambda}{4}$ point.

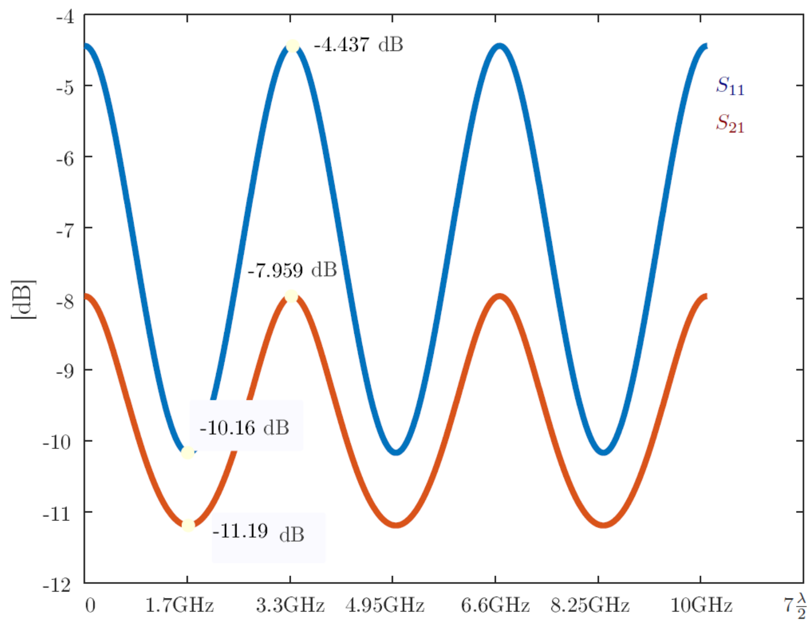

Figure 13: Simulation of 1/2 Wave Resonances Results

By performing an S-parameter sweep from 0 → 10 GHz we should see a resonance in the S-parameters, and furthermore we should see that resonance repeating at fixed $\frac{\lambda}{2}$ intervals as shown in Figure 13.

In the steady-state analysis I've shown, from the wave equation, how to calculate step by step the voltages for different wavelengths on a resonant structure. I've shown how easy it is to derive the S-parameters from the calculated voltages. Furthermore, I've derived the equation for the standing wave and its dependence of position. I've also proved that having a standing wave is not sufficient condition to develop dips/peak in the insertion loss profile of S-parameters. Additionally, I've shown that the resonance is happening at intervals multiples of half the wavelength, or in time, two times the propagation delay between the discontinuities, and that for a half-wave resonance to develop the discontinuities at both ends of the transmission line should be similar.

This has important practical implications as topologies of this kind can be found very often. In short, if you find a topology with at least two discontinuities with approximately the same value, separated by some transmission line length, and you see a dip/peak on the insertion loss, you should now be able to relate that resonance with the physical separation on the topology by calculating the resonance dips as indicated, and be equipped to understand the resonances observed in the S-parameters.

Now that we have introduced all the fundamental formulations, we can switch to the time domain. The next section provides a more visual and intuitive analysis of resonances and connects them with the fundamental analysis already completed in the frequency domain.

TIME DOMAIN ANALYSIS

Perhaps the easiest way to understand a resonance is from a time domain viewpoint. For example, we can create a simple simulation by sending a pulse on a transmission line that is reflecting part of the incident signal. Every time we introduce a transmission line, we need to recognize that voltages and currents will depend on two independent variables: time and space, meaning that $V(z_1, t_1)$ likely will be different than $V(z_2, t_2)$. As shown in the previous section we can expect to have traveling waves propagating in different directions in transmission lines. In the previous section, we formally analyzed the existence of traveling waves, in this section we will be looking at it at a higher level and more intuitively.

Single Reflection Case

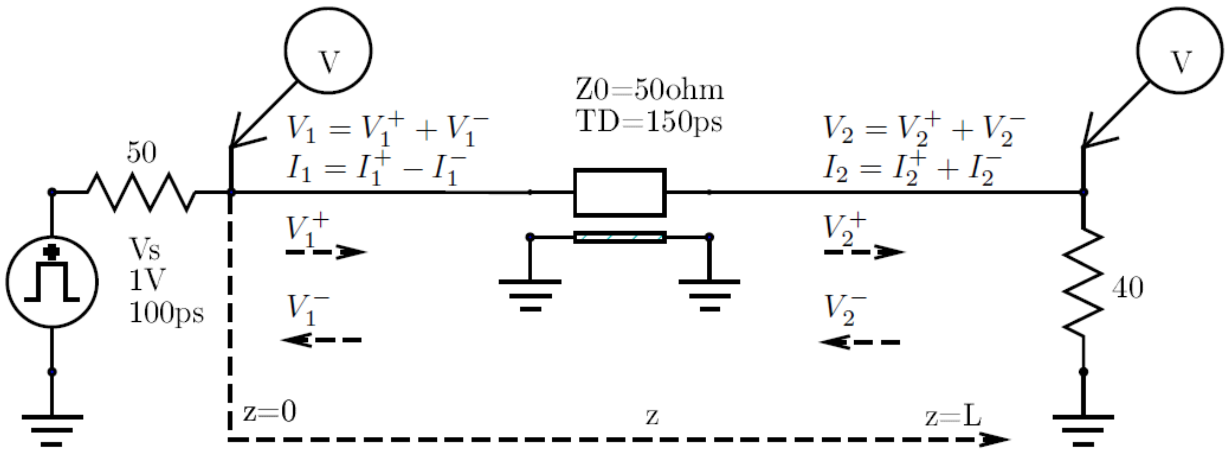

Figure 14: Time Domain, One Reflection Diagram

Figure 14 shows a simple circuit with $t_1 = tpd = 150\ ps$ lossless transmission line with a characteristic impedance of $Z_C = 50\ \Omega$. There is perfect matching at the source, and a slight reflection at the load, because $Z_L = 40\Omega$.

This circuit will have only two propagating waves. One is in the positive going direction from source to load, named $V_{in}(t)\ or\ V_1^+(t)$, and another on the negative going direction, from load to source, named $V_r(t)\ or\ V_1^-(t)$. Please note, in this nomenclature $V_{in}(t), V_1^+(t), V_1^-(t)$ happens at $z = 0$ and in the same way, $V_r(t), V_2^+(t), V_2^-(t)$ happens at $z = L$.

One important and sometimes easily forgotten point is that any voltage or current in the circuit, as it would be measured by an oscilloscope, is really the superposition of all the traveling waves passing by that location (z) and at that instant of time (t) as shown in 36

$$V(z=0, t=0) = V(0,0) = V_1(0) = V_1^+(0) + V_1^-(0) \qquad 36$$
$$V(z=L, t=t_1) = V(L, t_1) = V_2(t_1) = V_2^+(t_1) + V_2^-(t_1)$$

In such a simple circuit we can easily calculate these independent traveling waves by calculating the reflection coefficient at the load and source.

$$V_1^+(0,0) = V_s \cdot \frac{Z_0}{Z_0 + R_S} = 0.5V \quad (37)$$

$$V_1^+(0) = V_2^+(t_1) \quad (38)$$

$$\Gamma_L = \frac{V_2^-(t_1)}{V_2^+(t_1)} = \frac{Z_L - Z_0}{Z_L + Z_0} = -\frac{1}{9} \quad (39)$$

Please note Equation 38, which is defined as the continuity equation. It simply means that in the lossless environment, the propagating waveform sent is unchanged as it travels through the transmission line in either direction, forward or backward propagating wave. With the set of equations above, we can calculate the reflected voltage, and, thanks to the continuity equation, we can transfer it to the beginning of the line as follows:

$$V_1^-(2t_1) = V_2^-(t_1) = \Gamma_L \cdot V_2^+(t_1) = -55mV \quad (40)$$

Notice in 40 the reflected waveform of $-55mV$ will be arriving at the source with a delay of $2 \cdot tpd = 300ps$. Because our pulse source is only $100ps$ long, the reflected pulse will arrive to the source much after the injected pulse falls to zero. Hence the source will drive two steps before the reflection comes back, one from $0V\ to\ 1V$ and then $100ps$ later another step from $1V\ to\ 0V$. In essence, there will be two forced incident steps in this circuit, separated by $t_s = 100ps$.

The results of the simulation can be seen in Figure 15. The plot shows total voltages as you would see when measuring with an oscilloscope. We can observe the $500mV$ source or incident voltage, then after a propagation delay of $150ps$ how the voltage is slightly reduced at the load due to a negative reflection of $-\frac{1}{9}$. We see then the second step at the source from $1V\ to\ 0V$, and how it propagates to the load, finally observing the negative reflection pulse back at the source after the source has been quiet.

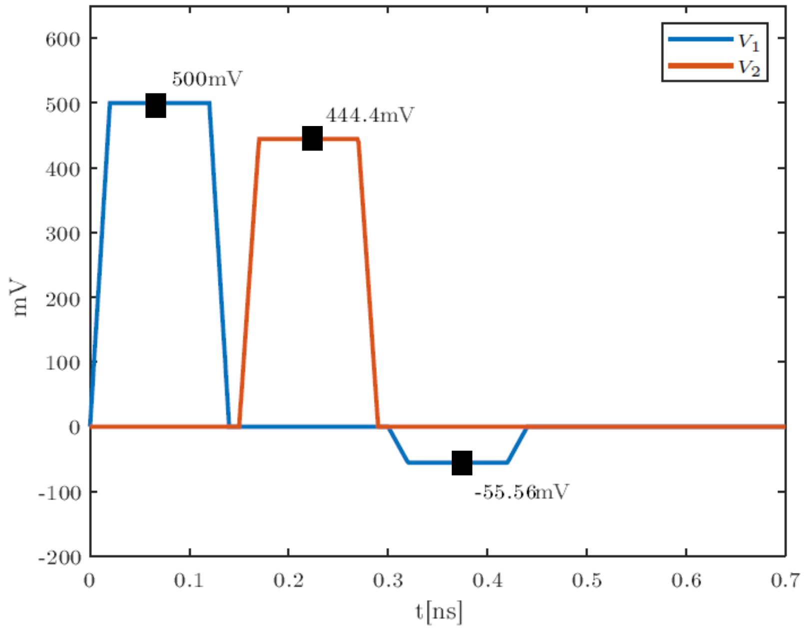

Figure 15: Single Reflection Results

Total voltage (as measured by an oscilloscope) is calculated as the superposition of traveling waves at a particular location and instant in time, under those conditions we can simply calculate them as follows:

$$V_2(t_1) = V_2^+(t_1) + V_2^-(t_1) = 444mV \qquad 41$$
$$V_1(2t_1) = V_1^+(2t_1) + V_1^-(2t_1) = -55mV$$

The nice feature of this scenario is that since we have only a single reflection at the load, but a perfectly matched source, in the result plot we can see directly the reflected propagating voltage $V_1^-(2t_1) = -55mV$ even though we are measuring the total voltages, since $V_1^+(2t_1) = 0$

Another observation is that for each incident pulse, we have a single reflection. A single propagating wave pulse travels in the "z+" direction and a single reflected propagating wave pulse travel in the "z-" direction. Loosely, in this context, we can conclude there are no resonances as expected from the lack of a second discontinuity as described in the resonance definition. We can see that on Figure 15, after the reflection is absorbed at the source, everything is quiet, "nothing resonates."

Dual Reflection Case

Let us now make the circuit resonate.

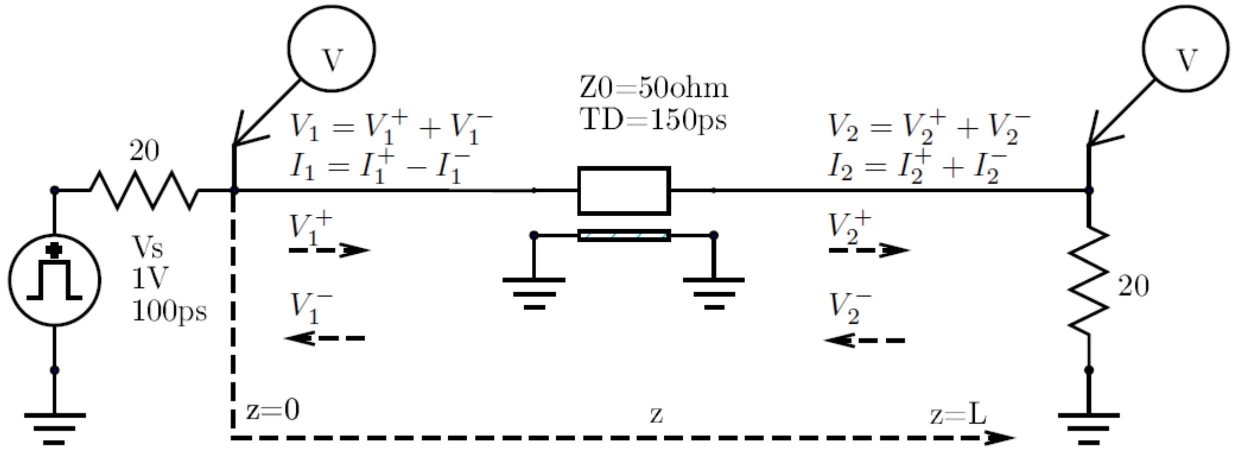

Figure 16: Topology with two Discontinuities

In this case, the requirement for a resonance to occur is met since we have two discontinuities separated by a transmission line. To see a bigger effect, we've increased the reflection at both ends by changing the source and load terminations to 20Ω

$$V_1(0) = V_1^+(0) = V_s \cdot \frac{Z_0}{Z_0 + R_S} = 714.3mV \quad (42)$$

$$V_1^+(0) = V_2^+(t_1) \quad (43)$$

$$\Gamma_L = \frac{V_2^-(t_1)}{V_2^+(t_1)} = \frac{Z_L - Z_0}{Z_L + Z_0} = -\frac{3}{7} \quad (44)$$

$$V_1^-(2t_1) = V_2^-(t_1) = \Gamma_L \cdot V_2^+(t_1) = -306mV \quad (45)$$

$$V_2(t_1) = V_2^+(t_1) + V_2^-(t_1) = 408.3mV \quad (46)$$

$$V_1(2t_1) = V_1^+(2t_1) + V_1^-(2t_1) + V_1^-(2t_1) \cdot \Gamma_S = -174.9mV$$

As shown above, all the reflections and ultimately voltages can be calculated. Note now how the reflected voltage calculation at the source gets a bit more complicated due to the un-matched source impedance.

A single pulse is sent, after its pulse width, **there is no more energy sent by the source**, nevertheless, when we observe the response, a clear resonance that continues over time long after the source pulse has ended is present as shown in Figure 17**Error! Reference source not found.**. The period and frequency of that resonance can be readily computed from the waveform.

This type of analysis is easily visualized using a lattice diagram as shown in Figure 18. The horizontal dimension represents space, the vertical dimension represents time. The voltage is calculated when a wave arrives at a port. This can be viewed, as sampling the continuous time domain voltage when the waveforms arrive at either port. Each arrow from source to destination and vice-versa represents the traveling wave from one port to the other. Reflected waves either at the source or destination are calculated as $V_{in} \cdot \Gamma_P$.

In addition, since in our example we have a pulse, not a step, there are two sets of traveling waves, one generated by the rising edge and the other by the falling edge of the pulse separated by a pulse width pw.

Those voltage sequences have been illustrated with different colors in the lattice diagram. Finally, every voltage at every port $V1, V2$ are calculated by summing the previous voltages at the line plus all the reflected and incident voltages leaving or entering a port.

Calculating every voltage following the lattice diagram is tedious and prone to error. To mitigate this problem, a closed form solution has been derived to calculate the sequence of reflections from a pulse as depicted in 47**Error! Reference source not found.**. The equation has been derived for V_2 , where n represents the reflection number at the destination port.

In our example, a pulse is sent with a rising and falling edge, because of it, the calculation of $V_2(n)$ must be done twice, for the rising and falling edge $+V_a$, $-V_b$ respectively.

After the series of reflection voltages for each n is calculated for V_2 rise and V_2 fall. The composite and final voltage is computed by $V_{2rise}(n) + V_{2fall}(n + 1)$. V_{2fall} $(n + 1)$ represents the voltage series shifted one tpd which is approximately the pw (pulse width) in our case.

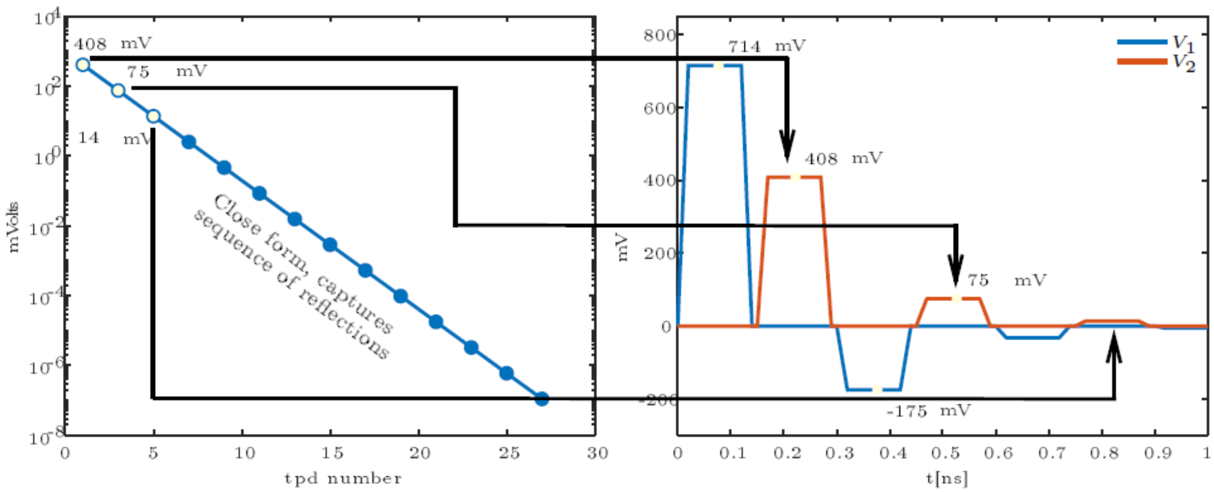

Figure 17: Closed Form for Attenuated Reflections

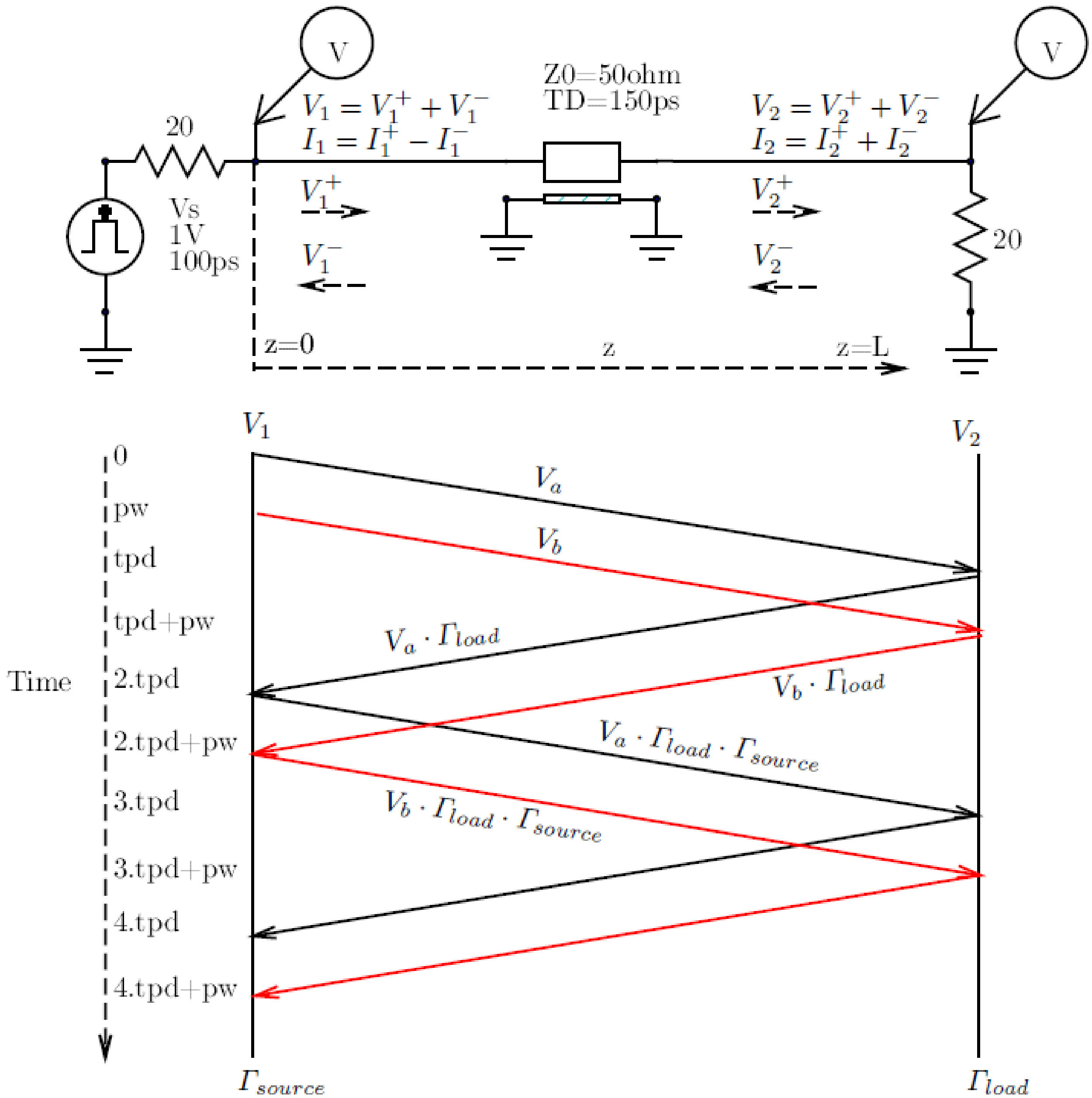

Figure 18: Lattice Diagram

$$V_2(n) = V_2(0) + V_a \cdot (\Gamma_L \cdot \Gamma_S)^{n-1} \cdot (1 + \Gamma_L) \qquad 47$$
$$V_2(0) = 0, V_a = V_S \cdot \frac{Z_0}{Z_0 + Z_C}, n = (1, \infty)$$
$$V_2(n) = V_a \cdot \frac{1 + \Gamma_L}{\Gamma_L \cdot \Gamma_S - 1} \cdot ((\Gamma_S \cdot \Gamma_S)^{n-1} - 1)$$

The results from the closed form solution to calculate reflections and attenuated voltages over time can be seen in Figure 18**Error! Reference source not found.**. Note the attenuations becomes so small, that a logarithmic scale is shown on the y-axis.

In our example, since we have the same discontinuities at the ends, the period $T = 300ps$ of the resonant frequency represents a round-trip delay between source and destination.

For a half-wave resonance to develop two additional conditions are required:

1. **The discontinuities at both ends of the transmission line should be the similar** In the extreme case, when the line is tied to ground at both ends, the voltages will be equal to zero since the line is grounded at those locations. This does not mean, the voltages elsewhere in the line will be also zero. The voltages along the line are the superposition of traveling waves at every point. This superposition of traveling waves produces a standing wave. In our example, and for this kind of standing wave, we will see a maximum absolute voltage at the middle of the line that will decay symmetrically as we approach the ends. Please realize this is **just one extreme case of a ½ wave** resonance**,** the other extreme is when the transmission is left open with a $\Gamma_S = \Gamma_L = 1$ .In this case, the resonance is such that the maximum absolute voltage will be at the ends, and the minimum will be at the middle. This resonance is just 180° off from the case in our example.
2. **The reflection coefficients at the ends should not be zero:** In order to have a discontinuity, we need reflection at the ends, the bigger the reflection, the bigger the resonance magnitude. The termination (discontinuity) values must be different from the transmission line characteristic impedance

Deriving a Half-Wave Resonance in the Time Domain

Steady state sinusoidal analysis was shown in the previous section to derive resonance patterns for different discontinuities leading to the concept of a standing wave and VSWR. In the time domain at a high-level we can perform simple tricks to show the same behavior.

Since we want to observe the voltages along the transmission line, the simulation deck will divide it to ten equal transmission line pieces with a $tpd_{piece} = \frac{tpd_{total}}{10}$ providing ten observation points along the transmission line as shown in Figure 19.

To make the reflections strong, the source and load terminations will be changed to 1Ω . This setup is close to the extreme shorted case at both ends but still allows us to inject a pulse into the circuit. The voltages at each location on the transmission line will be calculated and plotted so you can see the shape of the standing wave at different locations of the line.

Since we are trying to do a steady state analysis, but we have a transient pulse, we will compute the absolute value of the voltage at each location and calculate its mean. Ultimately, we'll end up with 10 points along the line with a mean voltage value. If we claim to have a ½ wave resonance, with very low source and load terminations as compared to the transmission line characteristic impedance, we should expect a maximum voltage at the center $\frac{L}{2}$ and minimum voltages symmetrically at the source and destination points.

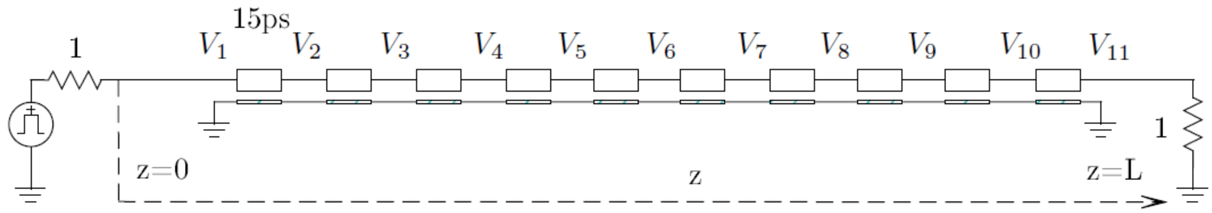

Figure 19: Time Domain Half Wave Resonance Diagram

by going from Step1 to Step3 we show the expected behavior. Remember, in our topology example with low source and load termination values with respect to the transmission line characteristic impedance, the reason we have a maximum of voltage at the center of the transmission line is because forward and backward traveling waves are adding constructively to form the maximum mean voltage on the line. At other locations of the line the superposition is not completely constructive, and finally at the end it is 100 percent destructive, meaning, at the end points where the wave hits either the source or load terminations, there is a reflection coefficient $\Gamma \cong -1$, meaning $V = V^+ \cdot (1 + \Gamma) \cong 0$. When we plot the voltage values in Step 3 and compare it to a sinewave, the resonance formed in the transmission line looks like half of a sinusoidal wave.

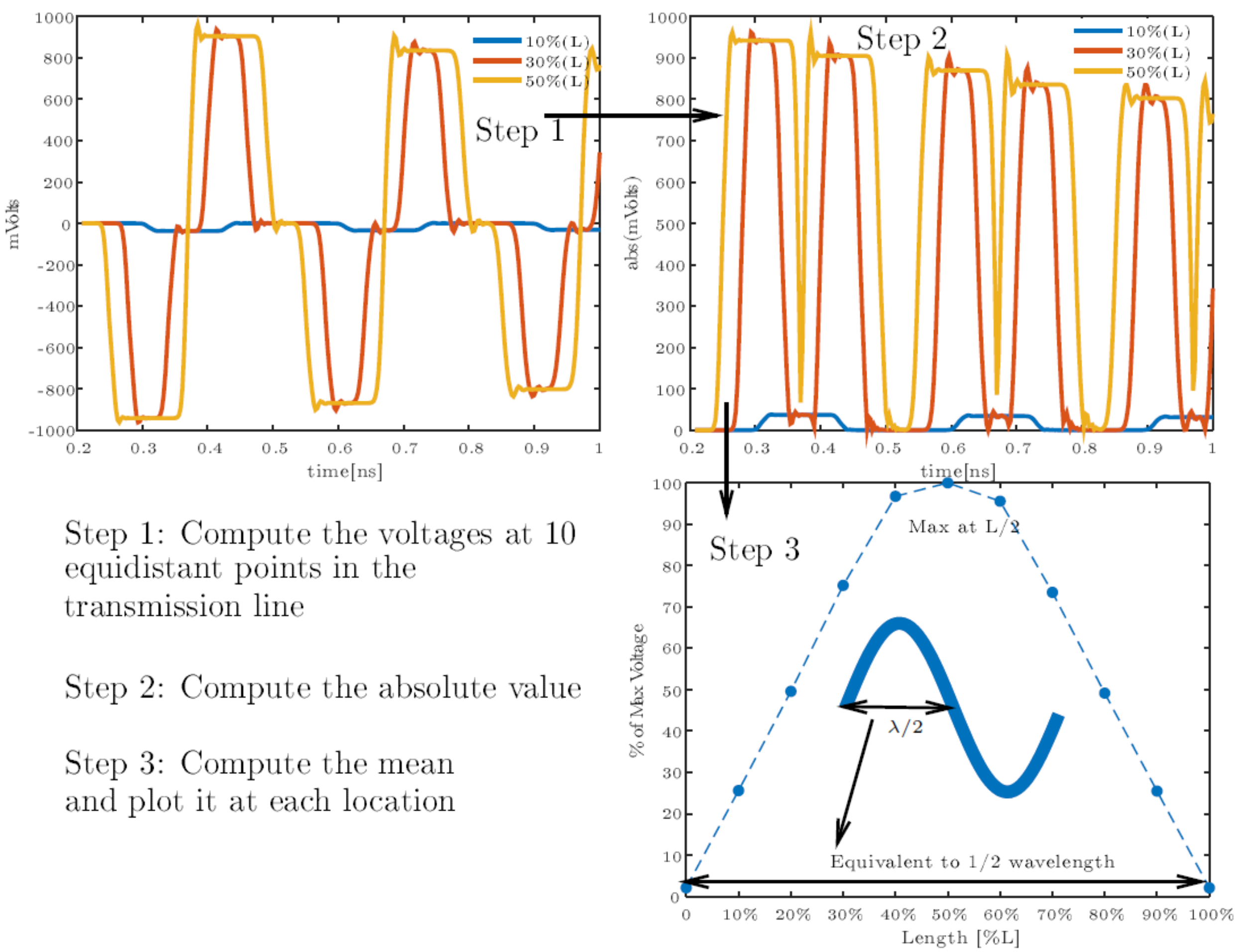

Figure 20: Half Wave Resonance Result in Time Domain

I wanted to touch base briefly on ¼ wave resonances that are also important and commonly found on many topologies. To accomplish this, I'll modify the topology slightly as shown in Figure 21, please notice the source termination is smaller, and the load termination is much higher than the transmission line characteristic impedance. Under those conditions a ¼ wave resonance will be formed.

In such a resonance we could expect the standing wave will look like ¼ of a sine wave, experiencing a minimum at the source and a maximum at the load.

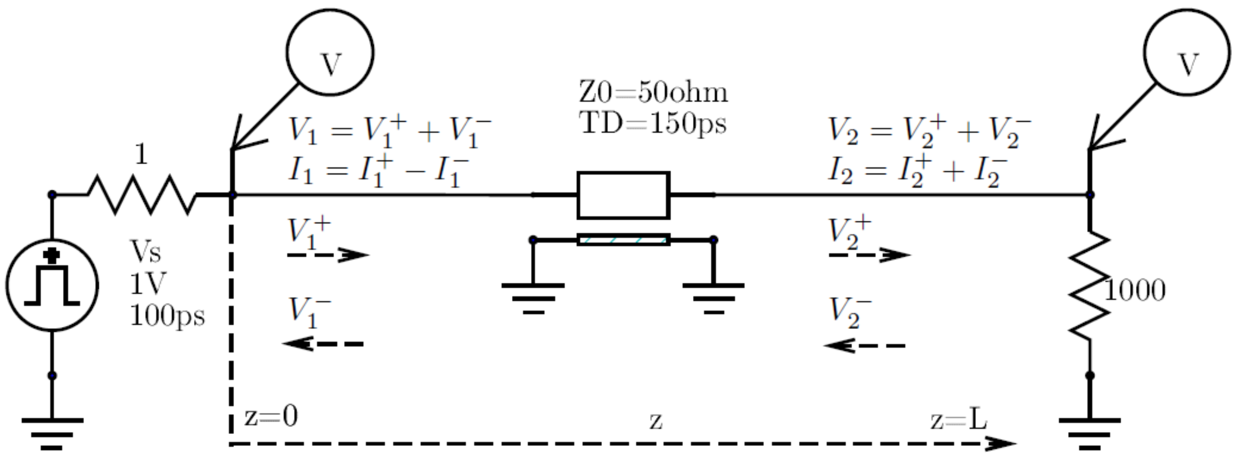

Figure 21: Quarter Wave Resonance Diagram

In Figure 22 it on the left, you see how the resonance period is $T = 600ps \ = 4 \cdot tpd$, meaning the traveling wave should travel back and forth twice for the resonance to develop. Also, using the same tricks done for the ½ wave, when plotting the voltage waveforms at all the points in the line, we can indeed see how the maximum voltage is developed at the end and the minimum is developed at the beginning. This type of structure in the RF literature is sometimes referred as a ¼ wave transformer and has the property of changing the phase of impedance by $180°$. It could transform an open at the end to a short at the beginning of the line.

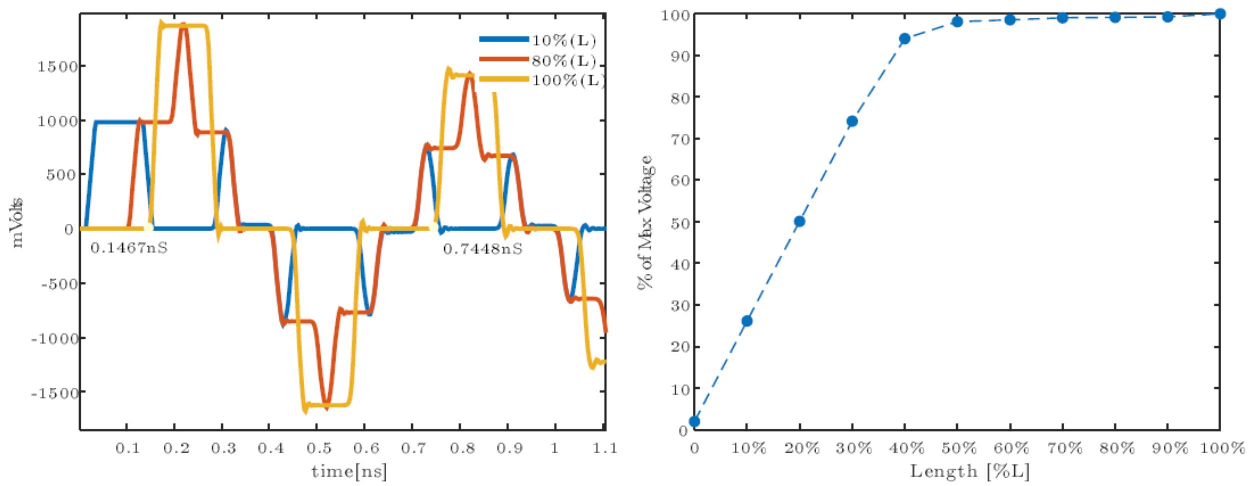

Figure 22: Quarter Wave Resonance Results

There is a very practical structure in signal integrity that behaves this way: via stubs. A stubby via in a 50Ω environment will see low impedance at one end, and an open stub at the other. If we consider the length of the stub as a transmission line, this forms a very strong ¼ wave resonator at high frequencies. If the electrical length of the via (at higher frequency) is long enough we could expect a strong resonance dip at that frequency and almost full attenuation.

We can clearly see these half-wave resonances both in the frequency and time domain, just a different approach to see the same effect. Ultimately, the point is that when we have these types of structures, we should expect to see dips/peaks at exactly:

$$f = \frac{1}{2 \cdot tpd_{line}} \qquad (48)$$

Well, exactly?

If you were observant, you might have noticed in prior sections, the discontinuities at the ends were always resistive. This is an interesting theoretical case since it allows us to develop all the conceptual framework and understanding, but, of very little practical applicability. Most of the time, at the ends we will encounter reactive discontinuities, in which case the peaks/dips will not follow exactly Equation 48, but for the most part it will be close.

Before we go into a couple of practical application cases, I'd like to explore some differences between resistive and reactive discontinuities when producing half-wave resonances.

Half Wave Resonance with Reactive Discontinuities

So far, all the termination discontinuities we used have been resistive. Let us compare a resistive to a capacitive discontinuity, to determine what happens. Figure 23 shows the topology for both cases.

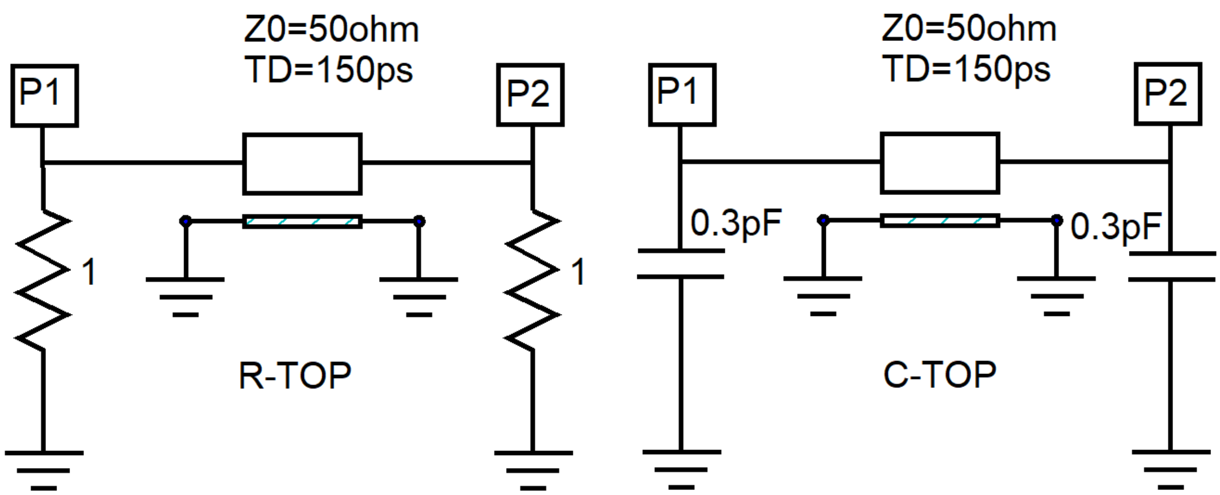

Figure 23:Capacitor vs. Resistive Discontinuity Diagram

We see something quite interesting and telling when computing both cases. Figure 24 shows the resistive and capacitive discontinuities. By following the markers, we realize that the first resonant frequency on the resistive case is indeed $f = \frac{1}{2 \cdot tpd} = 3.33GHz$ as expected, and it's shown as a peak in the insertion loss. Every other harmonic resonance is exactly $3.33GHz$ away from the fundamental and repeating with the same cadence.

On the other hand, for the capacitive discontinuity, we see the first resonance is close to the expected $3.3GHz$ but not exactly. Also, the first harmonic is shifting from the expected $f_{h1} = 2 \cdot 3.33GHz$. Furthermore, in the capacitor case we see a dip in the insertion loss as compared to a peak on the resistive case.

There are three questions we still need to answer, even for these simple cases:

1. Why does the resistive case have a peak and the capacitive case a dip in the insertion loss at the first resonant frequency?
2. In the capacitive case, why does the frequency of the dip not follow exactly this equation , $f = \frac{1}{2 \cdot tpd}$ as the resistive case does?
3. Why does the separation between harmonics seems to be changing on the capacitive case?

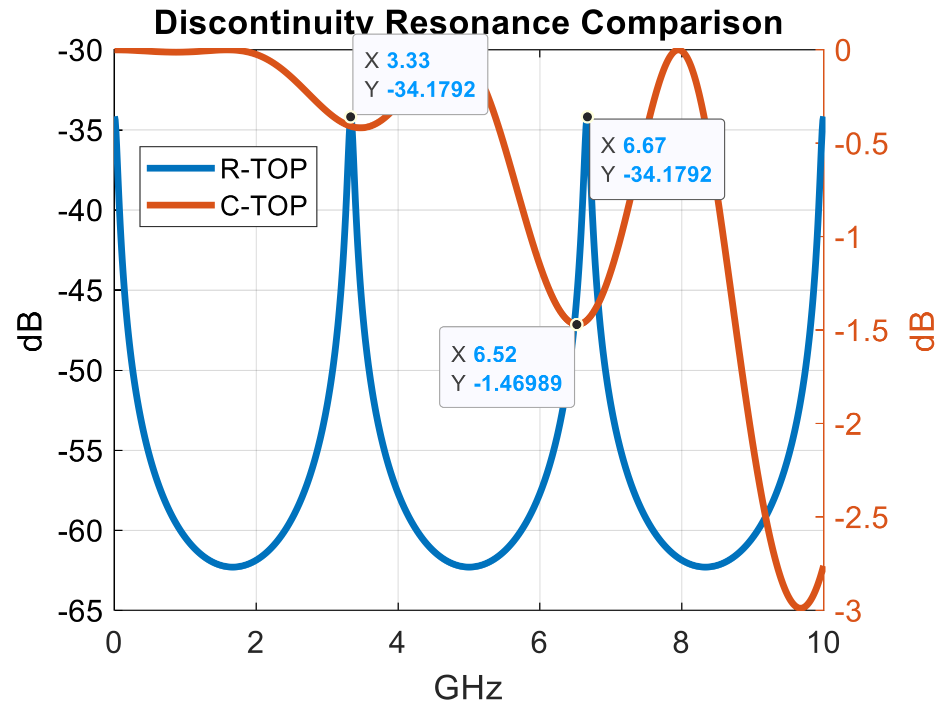

Figure 24: Capacitor vs. Resistive Discontinuity Results

From a simple and graphical viewpoint, the best way to explain the questions above is with the help of a phasor representation as shown in Figure 25

In the figure we see a V_{in} phasor plus another six phasors that means:

$\boldsymbol{V_{in}}$: Incident waveform with an angle of $0°$

$\boldsymbol{V_{R1}(R)}$: Resistive case, first reflection. Please note since $\Gamma \sim -1$, there is almost a full negative reflection with an angle of $180°$

$\boldsymbol{V_{R2}(R)}$: The first negative reflection bounces back to the source. At the source it'll have another negative reflection (since the termination at source and load is the same), hence bringing back the phase to $0°$.

If we pause a moment, and look at the diagram, we realize $V_{R1} and\ V_{R2}$ both rotate $180°$ resulting in a V_{R2} in phase with V_{in} allowing the maximum voltage to develop.

If you want to double check, you can refer to Equations 26 through 35, and you'll be able to reproduce exactly the peak amplitude in dB shown in Figure 24.

Furthermore, any other frequency producing a wavelength different from $\frac{\lambda}{2}$, will invariably result in lower insertion loss values. This can be visually understood realizing that $V_{R1} and\ V_{R2}$reflections will not perfectly cancel, and the resulting vector will ultimately subtract from the incident voltage. You can also check this out but going through the equations. I'll leave this exercise to the reader.

$\boldsymbol{V_{C1}(XF_1)}$: Represents the first reflection at the fundamental on the capacitive case. With a capacitive discontinuity we realize that the reflection does not have a phase of $180°$ as in the resistive case, but only a phase of approximately $-90°$. The reflection is not exactly $-90°$ since the capacitor is loaded with the parallel impedance of the transmission line and port impedance and that changes the phase.

$\boldsymbol{V_{C2}(XF_1)}$: Represents the second reflection at the fundamental frequency, and as explained above produces another rotation of $-90°$, bringing the total phase rotation to approximately $-180°$.

We can immediately see, different from the resistive case the full rotation will produce a phasor that is approximately (but not exactly) $180°$ from V_{in}, in essence subtracting from it and producing a minimum in the insertion loss, not a maximum.

$\boldsymbol{V_{C1}(XF_2)}$: Represents the first reflection on capacitive case at the first harmonic (higher) frequency. We need to realize that as the frequency increases, the capacitive discontinuity becomes closer and closer to a short, the phase gets closer to what it was with the resistive discontinuity.

$\boldsymbol{V_{C2}(XF_2)}$: Represents the second reflection on the capacitive case at the first harmonic (higher) frequency. We can see now that at higher frequencies, the second rotation gets closer to the resistive case and hence at higher frequencies the peaks and dips between these two cases will align as shown in Figure 26.

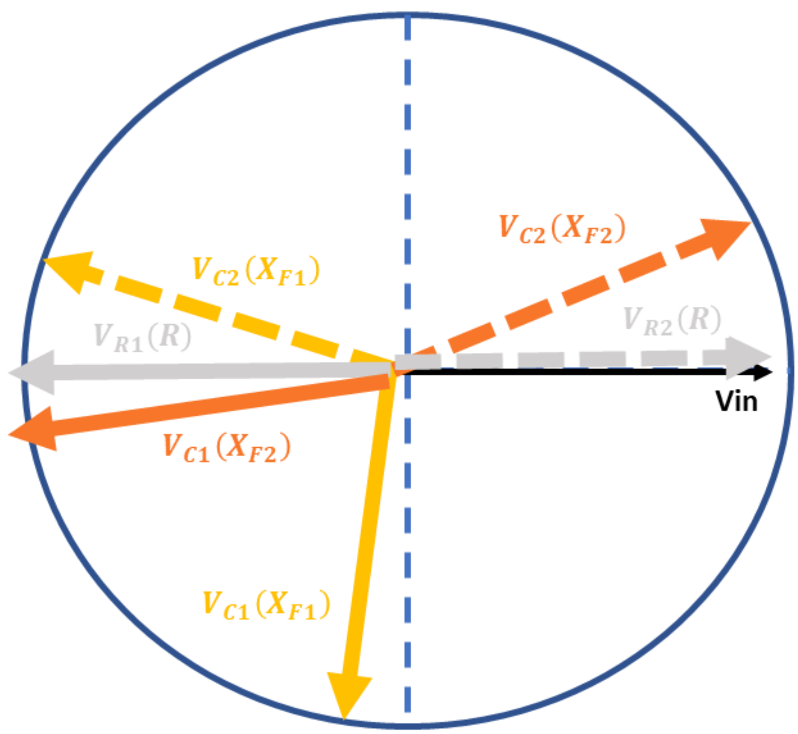

Figure 25: Phasor Representation of Capacitive and Resistive Discontinuities

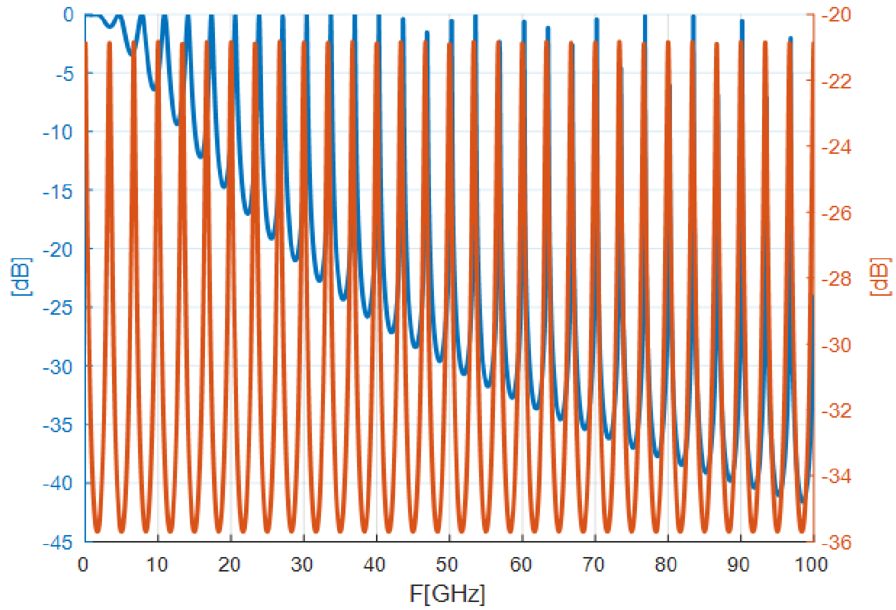

Figure 26: Resonance Shift on Capacitive Case

With the above examples we've been able to explain some subtleties of different types of discontinuities from a high-level visual viewpoint. But at this point with all the knowledge gained so far, we are in a good position to look at a couple of practical examples.

Practical Examples Of Half-Wave Resonances

In this section I'll present two cases that are very common for the SI-engineer to encounter in several topologies. I'll show a cable assembly example and an open pin field connector.

You'll see these two structures are particularly susceptible to half-wave resonances.

Cable Assembly

A cable assembly is a cable connecting two circuits. For high frequency applications in particular, cables are selected to minimize as much as possible the losses, but this makes the structure more susceptible to resonances. A cable assembly will contain the cable and the termination at both ends, where the cable connects either to a connector or directly to a board. Most often than not, those end cable connections are the same at both extremes of the cable and as much as people try to minimize those connecting discontinuities they can't completely be eliminated.

Figure 27 shows an example of a short cable assembly

Figure 27: Cable Assembly Example

I will construct the model of a generic cable assembly, with different lengths and realistic discontinuities at each side, and we'll study the insertion loss to determine if we can see the resonance and how several parameters affect it. Please note the end discontinuities could come from multiple sources. For example, when we peel a cable to solder into a board or connector directly or a breakout region with capacitive vias at the end, or any other number of small imperfections in the channel that results from having to somehow, connect the cable to the end points.

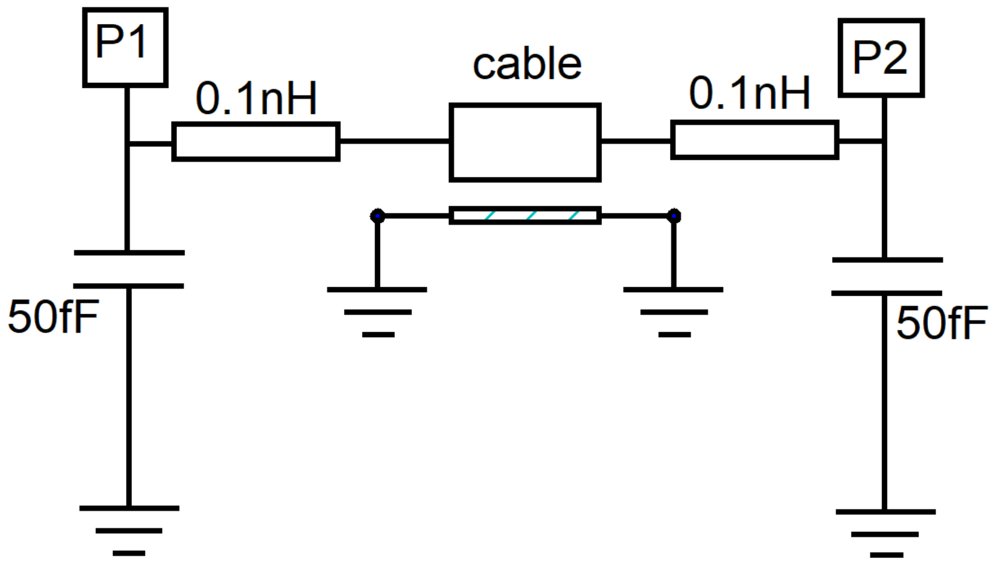

Figure 28: Cable Assembly Diagram

For this test structure, I used a good cable, and created reasonable end discontinuities at each end to compute the insertion loss for different lengths. In Figure 29 you can see the TDR for different lengths. As expected for the longer cables, the discontinuity blip at the end looks reduced due to the normal attenuation loss of the cable.

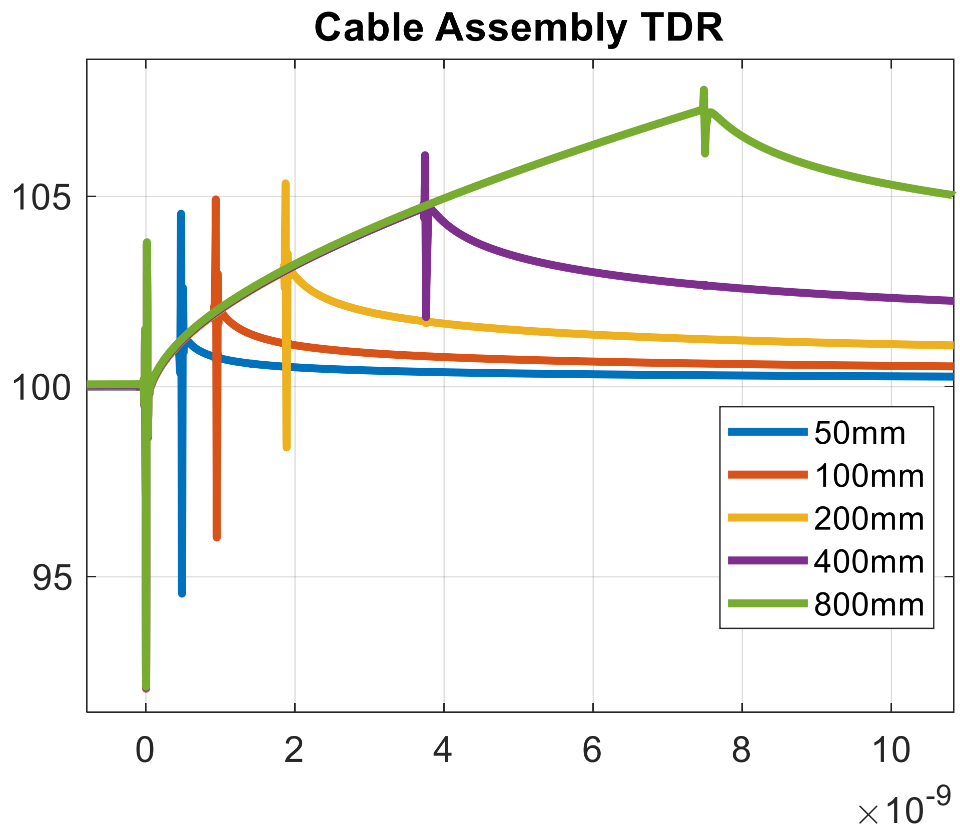

Figure 29: Cable Assembly TDR

When we look at the insertion loss profile in Figure 30, we can clearly see the half-wave resonance on the cables. We can see that the shorter the cable, the less loss it has, the wider the fluctuation of the resonance, and the more you see it in the profile. As losses increase, and the fundamental resonance decreases, the ripples are less evident as can be seen by the green curve.

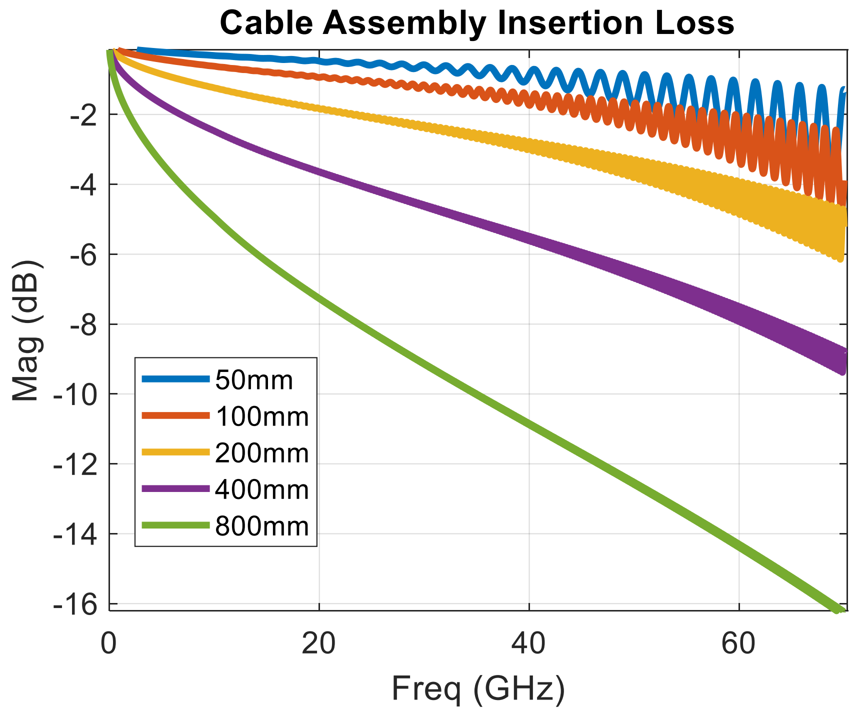

Figure 30: Cable Assembly Insertion Loss

This is very typical so you should not be surprised when you see this type of behavior, particularly in shorter cable assemblies.

Open Pin Field Mezz Connector

In this case, we have a short mezzanine connector used to attach two boards. These connectors come in many shapes, flavors, and heights.

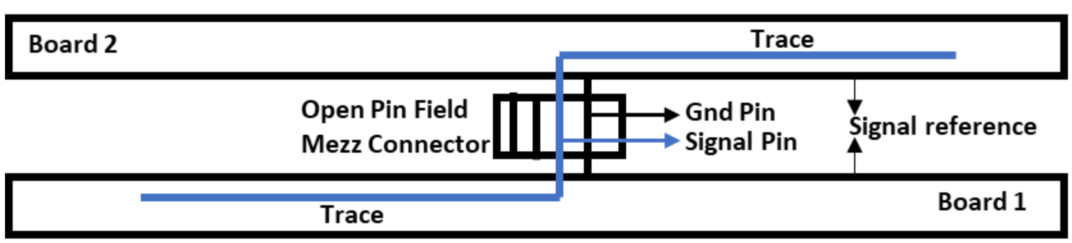

Figure 31: Mezzanine Connector Diagram

Many high frequency connectors, particularly for very high frequencies, contain special structures in order to carry the GND signals from one side to the other, basically making the pins locations defined a priori. An open-pin field connector is a connector that treats every pin equally. This means, it's up to the user to select which pins connect to signals and which pins connect to gnd. Many of these open pin-field connectors are very flexible, since it allows engineers to define their own pin-out and if properly designed into systems, they would allow the transfer of high-frequency signals as well.

In this example, when you look at Figure 31, please note I have included two different pins. One is a signal pin, but the other is a GND pin.

Even though in the signal pins we can't always guarantee that the impedance inside the connector will be the same as the impedance on the surrounding breakout regions on the boards, we could expect the delta in impedance not to be big. On the other hand, when we press attention to the GND pin, we notice we have a situation very similar (almost identical) to the R-TOP of Figure 23. The pin has some finite impedance inside the connector, but it is tied to big planes and GND so that the pin is shorted at both ends.

So even though the height of the connector in general is short, we should expect to see a resonance effect.

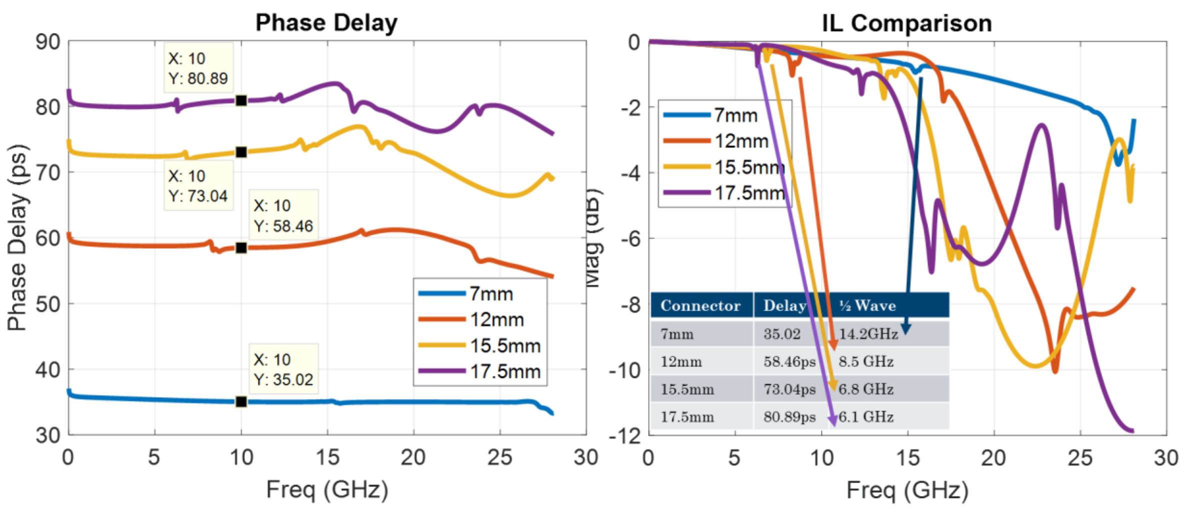

Figure 32: Mezz Connector Insertion Loss Results for Different Connector Heights

In Figure 32, we can see the dips clearly changing frequencies as the connector height changes. By computing $f \cong \frac{1}{2 \cdot tpd(height)}$ we can see the calculations predict very closely the dips, proving the resonance is a half-wave resonance depending on the connector height.

Another important aspect to consider is that a resonating structure can make a great antenna. At the resonant frequencies we could expect to have more crosstalk. In this situation, by doing a field solver simulation and computing the fields before, at and after the resonance frequency, we can clearly see the effect of it as illustrated in Figure 33.

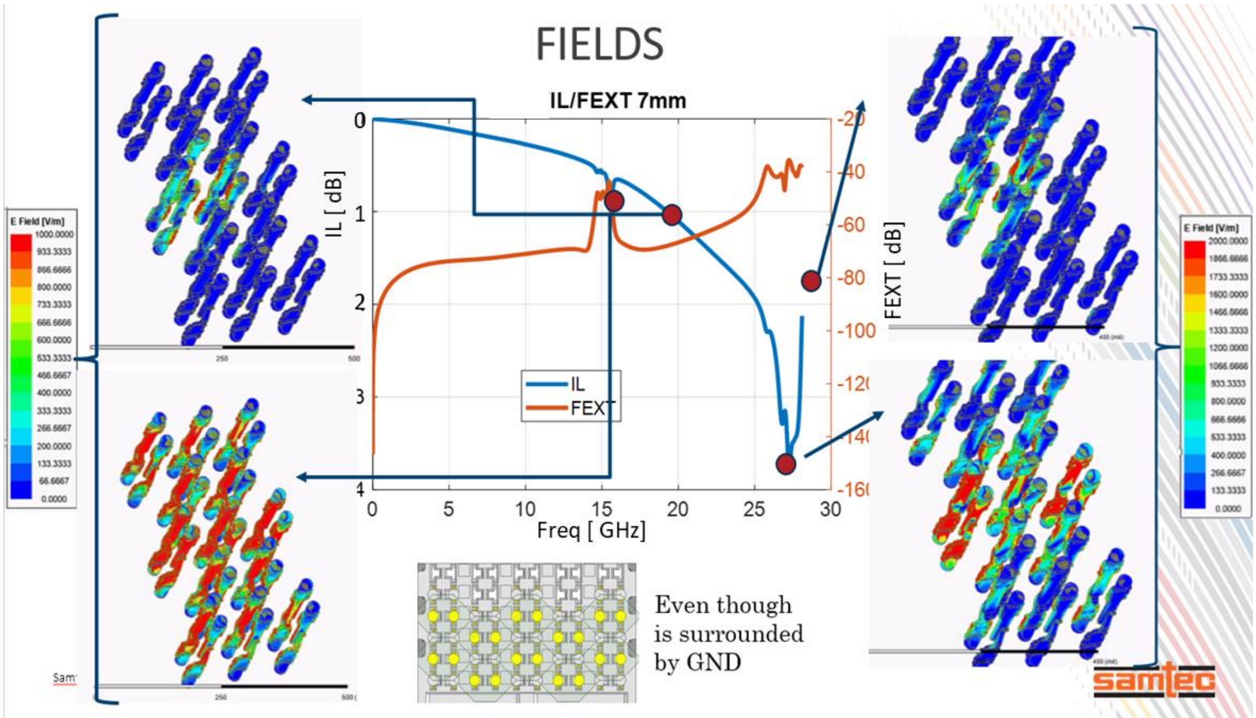

Figure 33: Resonant Fields and Cross Talk

In conclusion, you can see how easily and readily, half-wave resonances can be found in typical topologies. Pretty much any component you insert into a system, will likely be attached similarly at both ends in which case you are promoting this type of resonance.

By understanding the mechanism of the resonance and realizing how to identify and calculate its resonant peaks/dips, you can attempt to minimize its effect.

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